Calc II - Prep for the final exam

Our midterm will be this Friday, July 16. This will be an *in class* exam and you'll have the full class period to do it.

The final will be you, paper, and pencil. Calculators will not be needed or permitted.

The structure might look something like the following, though the problems will certainly be different.

1. Evaluate the following integrals.
   (a) $\displaystyle\int x\sqrt{9-x^2}\,dx$
   (b) $\displaystyle\int_0^3 \sqrt{9-x^2}\,dx$
   (c) $\displaystyle\int x\sin(2x)dx$
   (d) $\displaystyle\int_0^\pi \sin^3(x)\cos^2(x)\,dx$
2. Suppose we wish to estimate
$$\int_0^2 \sin(x)dx$$
with a midpoint sum and we'd like our result to be within 0.0001 of the actual value.
   (a) Find an n large enough so that n terms will guarantee your estimate is within the desired accuracy.
   (b) Write down the resulting sum using summation notation.
3. Compute $\displaystyle\int_1^\infty \frac{1}{x^3}\,dx$.
4. Suppose we spin the region under the graph of $f(x) = 1/x^3$, to the right of the line $x = 1$, and over the x-axis around the x-axis. What is the volume of the resulting solid?
5. Use u-substitution to express the following normal integral as a standard normal integral:
$$\frac{1}{3\sqrt{2\pi}}\int_0^5 e^{-(x-2)^2/18}\,dx$$

6. Evaluate $\displaystyle\sum_{n=2}^{\infty} 2\frac{(-4)^n}{3^{2n}}$.

7. Use the integral test to show that
$$\sum_{n=1}^{\infty} \frac{1}{n^4}$$
converges.

8. Write down a couple complete sentences using the alternating series test to show that
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$$
converges conditionally

9. Classify the following series as absolutely convergent, conditionally convergent, or divergent.
Note: You need not justify your assertion.
   (a) $\sum (-1)^n \frac{\sqrt{n}}{\sqrt{n^2+1}}$
   (b) $\sum (-1)^n \frac{\sqrt{n}}{\sqrt{n^3+1}}$
   (c) $\sum (-1)^n \frac{\sqrt{n}}{\sqrt{n^5+1}}$

10. Find the domain of convergence of the power series $\sum (-1)^n \frac{3^{n+1}}{n^2} x^n$.

11. Starting with the geometric series formula, find a power series for
$$f(x) = 1/(1+x^3).$$
Then integrate to find a series representation of
$$\int \frac{1}{1+x^3}\, dx.$$

12. What function does $\sum_{n=1}^{\infty} n^2 x^n$ represent?

13. Suppose we use a rope with a linear density of 2 kg/m to pull a 10 kg diamond up 20 meters. How much work is done in this process?