

MML - Review for Exam 2

We will have our second exam next Friday, February 20. This review sheet is genuinely meant to help you succeed on that exam.

Generally, I will expect *solutions* to the problems, as opposed to just answers. So, for example, if the answer to an optimization problem is $y = 5$, then the solution will consist of a clear explanation with correctly written supporting computations indicating *why* the answer is $y = 5$.

The problems

1. Write down definitions of the following:
 - Linear transformation
 - Linear span of a set of vectors
 - Linear independence of a set of vectors
 - Subspace of \mathbb{R}^n
 - Basis of a subspace

2. A bit of data on NBA players is shown in Table 1.
 - a. What are the cases in the data table?
 - b. Name one numerical variable.
 - c. Name one nominal, categorical variable.
 - d. Name one ordinal, categorical variable.

Table 1: NBA Players

first_name	last_name	team	team_abbr	position	number	height
Alex	Abrines	Thunder	OKC	Guard	8	78
Jaylen	Adams	Hawks	ATL	Guard	10	74
Steven	Adams	Thunder	OKC	Center	12	84
Bam	Adebayo	Heat	MIA	Center-Forward	13	82

3. Consider the numeric data $\{1, 1, 2, 4\}$.

a. Write down the computation showing that the mean is 2.

b. Write down the computation showing that the standard deviation is $\sqrt{3/2}$.

4. Compute the determinants of the following matrices and use that information to determine the singularity or non-singularity of each matrix.

a.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

b.

$$B = \begin{bmatrix} 1 & 2 & 0 & 8 & -1 & 0 & 6 & 10 \\ 2 & 0 & -8 & -1 & -1 & -1 & 1 & 1 \\ 0 & 3 & 3 & 12 & 0 & -1 & 0 & 2 \\ 1 & 10 & -6 & -25 & -1 & 0 & -1 & -4 \\ 3 & 3 & -74 & 4 & 5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & -1 & 5 & 1 & 0 \\ 4 & 0 & 1 & -4 & 0 & 1 & 0 & 1 \end{bmatrix}$$

c.

$$C = \begin{bmatrix} 1 & 0 & -4 & 2 & 1 & -9 & 1 & 3 \\ 0 & 1 & 2 & 1 & 0 & 0 & 17 & 0 \\ 0 & 0 & 1 & -4 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -72 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & -4 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

5. Suppose that the matrix A and its reduced row echelon form R are

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 5 & 7 & 9 \end{bmatrix} \quad R = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

a. Provide a basis for the column space of A .

b. Provide a basis for the null space of A .

c. Provide a basis for the range of A .

6. Show that the null space of a linear transformation mapping $\mathbb{R}^n \rightarrow \mathbb{R}^m$ is closed under linear combinations and is, therefore, a subspace of \mathbb{R}^n .

7. Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}.$$

- a. Row reduce the augmented matrix $[A|I]$.
- b. What is A^{-1} ?

8. Suppose that A is a non-singular matrix. Show that

$$(A^{-1})^T = (A^T)^{-1}.$$

9. Find a value of t such that

$$\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 2t \end{bmatrix} \quad \text{and} \quad \mathbf{y} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 3 \end{bmatrix}.$$

are perpendicular.

10. Find the projection $\text{proj}_{\mathbf{b}} \mathbf{x}$ of the vector \mathbf{x} onto \mathbf{b} , where

$$\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 2 \end{bmatrix}.$$

11. Find the least squares solution to the overdetermined linear system

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$