MML - Review for Exam 3

We will have our third exam this Friday, March 28. This review sheet is again meant to help you succeed on that exam.

Generally, I will expect *solutions* to the problems, as opposed to just answers. So, for example, if the answer to an optimization problem is $y = 5$, then the solution will consist of a clear explanation with correctly written supporting computations indicating *why* the answer is $y = 5$.

The problems

1. Write down a careful definition of each of the following.

   b. Eigenvalue/Eigenvector pair for a matrix A
   c. Similarity of matrices A and B
   d. Principle component of a data matrix X

2. Diagonalize the matrix
$$A = \begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}$$
   That is, express the matrix as a factorization $A = SDS^{-1}$ where D is diagonal. You do *not* need to compute the inverse of S explicitly.

3. Diagonalize the matrix
$$B = \begin{pmatrix} 1 & 1 & 4 \\ 0 & -2 & 0 \\ 0 & -1 & -3 \end{pmatrix}$$
   That is, express the matrix as a factorization $B = SDS^{-1}$ where D is diagonal. You do *not* need to compute the inverse of S explicitly.
   Comment: Maybe I'd give you the eigenvalues and eigenvectors for this problem??

4. Compute

$$A^{42}\begin{bmatrix}1\\1\end{bmatrix}$$

where A is the matrix in problem 2 and

$$B^{42}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

where B is the matrix in problem 3 and

5. Suppose that A is similar to B and that λ is an eigenvalue of A with corresponding eigenvector $\vec{x}$. Show that λ is also an eigenvalue of B. What is the corresponding eigenvector of B?

6. Bob says that every 3×3 matrix is similar to its own inverse. Provide a counter example showing that Bob is wrong.

7. Suppose that A, B and C are $n \times n$ matrices with A similar to B and B similar to C. Use the definition of similarity to prove that A is similar to C.

8. Consider the data matrix X given by

$\mathbf{x}_1$	$\mathbf{x}_2$
1	3
2	2
3	1

   a. The principal components of X are the eigenvectors of what matrix?
   b. In what direction does the first principal component point?
      It might help to draw a picture!