MML - Lead up to Exam 2

We will have our second exam Friday, Jan 21. Here are some problem to help whet your appetite. I will probably add a few more problems by our next class period and make a separate, proper review sheet over the weekend.

The problems

1. Define $U \subset \mathbb{R}^2$ by
$$U = \{\langle x, y\rangle \in \mathbb{R}^2 : 2x + 3y = 0\}.$$
Is U a vector space? If so, then prove that U is closed under linear combinations. If not then explain clearly why.
2. Define $V \subset \mathbb{R}^2$ by
$$V = \{\langle x, y\rangle \in \mathbb{R}^2 : 2x + 3y = 5\}.$$
Is V a vector space? If so, then prove that V is closed under linear combinations. If not then explain clearly why.
3. Let
$$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}.$$
Give a complete characterization of the range of the linear transformation $\vec{x} \to A\vec{x}$.
4. Suppose that the matrix A and its reduced row echelon form R are
$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 5 & 7 & 9 \end{bmatrix} \quad R = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
a. Give a complete description of the column space of A.
b. Give a complete description of the null space of A.
c. Give a complete description of the range of A.
5. Show that the null space of a linear transformation is closed under linear combinations.

6. Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}.$$

   a. Row reduce the augmented matrix $[A|I]$.
   b. What is A^{-1}?

7. Suppose that A and B are invertible matrices of the same size. Show that $(AB)^{-1} = B^{-1}A^{-1}$.

8. Suppose that the matrix

$$B = \begin{bmatrix} 0 & -2 & 2 & 4 & 10 \\ -2 & -3 & -1 & -1 & -1 \\ 0 & -1 & 1 & 4 & 1 \\ 2 & 3 & 4 & 7 & 4 \\ 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

is row equivalent to

$$D = \begin{bmatrix} 2 & 3 & 4 & 5 & 6 \\ 0 & -1 & 1 & 2 & 3 \\ 0 & 0 & 3 & 4 & 5 \\ 0 & 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

and that the row operations to get from B to D are

- R1<->R5,
- R1+R4 -> R4,
- R3+R4 -> R3,
- R2 - R1 -> R2,
- 2R3 + R5 -> R5
- R4 -> -R4
- R2<->R3.

What is det(B)?

9. Let V denote the vector space of continuous functions defined on $[0, 1]$. Show that the inner product

$$\langle f, g \rangle = \int_0^1 f(x)g(x)\,dx$$

preserves linear combinations.