Calc I - Challenge problem

Here's a problem that will *not* be on the rehash. I'll tell you soon why this kind of thing is worth thinking about, though. Let's think of it as a *challenge problem*.

The challenge problem:

Figure 1 shows the graph of

$$f_a(x) = \frac{1}{1+e^{-ax}},$$

where a is a real parameter set to $a = 1$ in the picture. We'd like to choose the parameter a to approximate the data points

$$\{(-1, 0), (-1/5, 1), (1/5, 0), (1, 1)\}$$

shown in the figure as well as possible. More specifically, we'd like the sum of the squares of the lengths of the four vertical line segments from the data points to their approximations on the graph to be as small as possible. Let's call that the *total squared error*.

To minimize the total squared error,

- Write down a function $F(a)$ that represents the total squared error as a function of the parameter a and
- Compute $F'(a)$.

Don't worry about solving $F'(a) = 0$! We'll talk about it later.

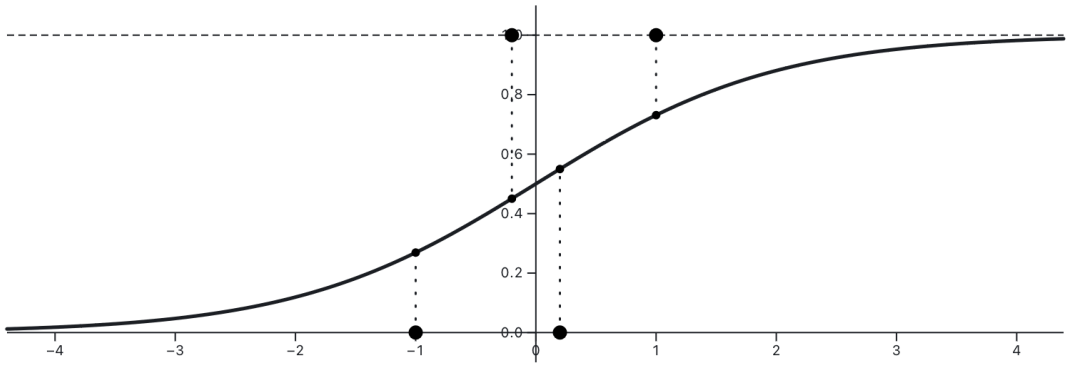

Figure 1: Picture illustrating the total squared error