Calc I - Review for Quiz 1

We have our first quiz this Friday, Jan 24. All the problems on that quiz will likely look like something you see on this problem sheet, though this sheet is a bit longer than the quiz will be.

1. Curious about the following limit,
$$\lim_{x \to 0} (1 + x)^{2/x},$$

   I used my computer to plug in several values of x that are *close* to 0 but *not equal* to 0. The results are shown in the table below.

x	0.1	0.01	0.001	0.0001	0.00001
$f(x)$	6.7275	7.31602	7.38168	7.38832	7.38898

   Based on those computations, can you make a conjecture as to the approximate value of the limit? Be sure to indicate how many digits you believe to be correct and why.

2. The graph of
$$f(x) = \frac{x - 1}{x^3 - x^2 + x - 1}$$

   is shown in Figure 1 below.

   a) Judging from the figure, what do you suppose is the value of $\lim_{x \to 1} f(x)$?
   b) Use a little algebra together with the limit laws to prove that your guess is correct.

3. The Complete graph of a function f is shown in Figure 2 below. At each of the points $a = -1$, $a = 1$, $a = 2$ and $a = 4$, find the value of

a) $f(a)$,
b) $\lim_{x \to a^-} f(x)$,
c) $\lim_{x \to a^+} f(x)$, and
d) $\lim_{x \to a} f(x)$.

4. Continuing with Figure 2, state one clear reason why f is discontinuous at each of the points $a = -1$, $a = 1$, $a = 2$ and $a = 4$.

5. Compute each of the following limits. For part (a) make sure to write your solution out carefully. I'm primarily interested in answers for the others.

a) $\lim\limits_{x \to 2} \dfrac{2x^2 - 3x - 2}{x - 2}$

b) $\lim\limits_{x \to \infty} \dfrac{2x^2 - 3x - 2}{x - 2}$

c) $\lim\limits_{x \to \infty} \dfrac{2x^2 - 3x - 2}{x^2 - 2}$

d) $\lim\limits_{x \to 2} \dfrac{x + 1}{x^2 - 4}$

e) $\lim\limits_{x \to 4} \dfrac{x + 1}{(x - 4)^2}$

6. Let $f(x) = x^3 - x^2 - x - 1$. Write a complete sentence explaining why there is a number $c \in (0, 2)$ such that $f(c) = 0$.
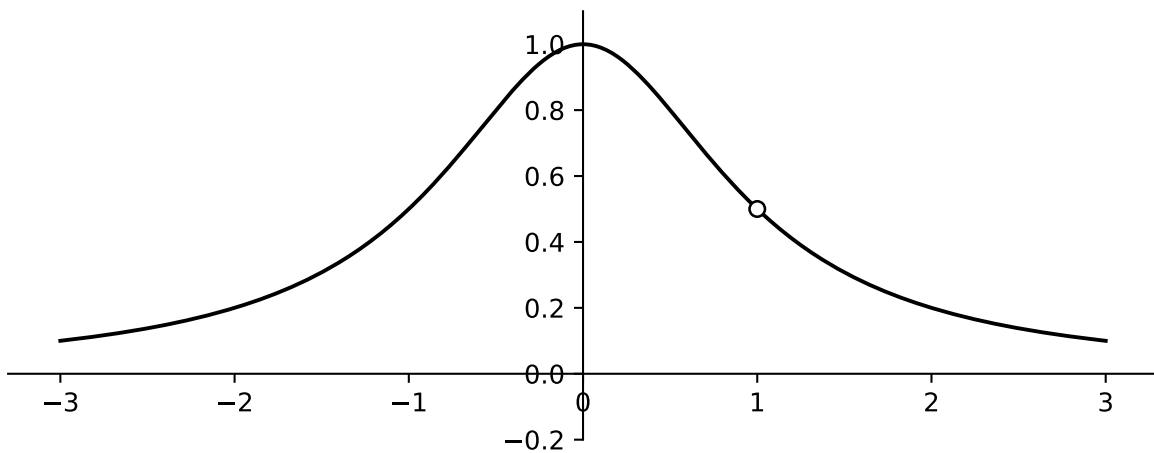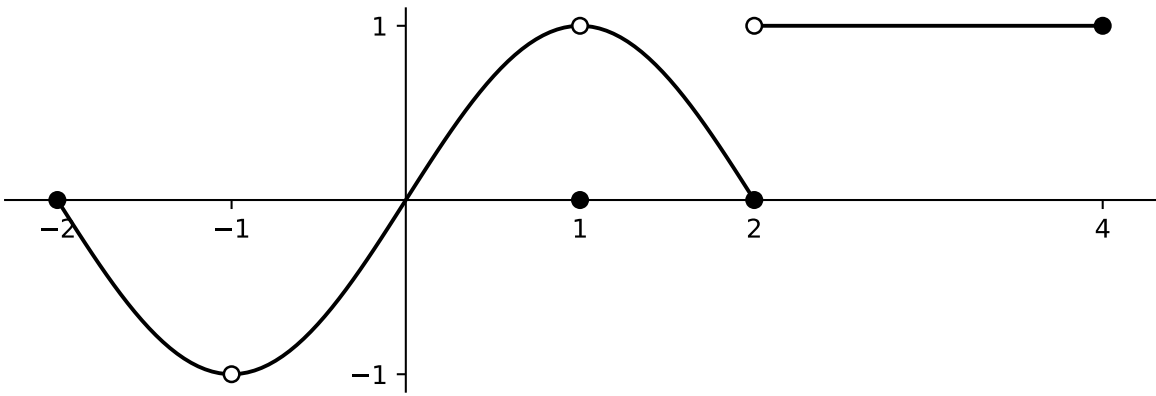

Figure 1: The graph of $(x - 1)/(x^3 - x^2 + x - 1)$

Figure 2: Figure for limits and continuity