Calc I

Review problems for the final exam

The Final exam will be during finals week

- on Friday, May 2nd at 8:00 AM for the 8:00 AM section and
- on Monday, May 5th at 8:00 AM for the 9:30 AM section.

This review sheet is a little different than our past review sheets. It consists of four sections: three sections containing exact problems off of past exams and review sheets and one more section containing problems on the probability theory that we've studied since the last exam.

Nonetheless, I will base the final exam on this review sheet, just as I have past exams. You should study it thoroughly because doing so will help your performance on the final. Do *not* just assume that you know the material since you did well on it before. Final exams can have a significant impact on final grades in my experience. Full letter grade shifts are not uncommon in my experience.

Note that the Math Lab will be open regular hours through finals week. You can also ask questions through the discussion link at the bottom of the web version of this document.

Exam 1

- **Exam 1, Problem 3**: Compute the following limits.

  a) $\displaystyle\lim_{x\to 2^+} \frac{x+1}{x-2}$

  b) $\displaystyle\lim_{x\to 3} \frac{x-3}{3x^2-10x+3}$

- **Exam 1, Problem 4**: Write down a complete sentence referring to the intermediate value theorem explaining why $f(x) = 3x^3 - x - 1$ has a root between $x = 0$ and $x = 1$.

- **Exam 1, Problem 5**: Find the derivatives of the following functions, using the differentiation rules we've learned in class.

  (a) $f(x) = x^2 - x$

  (b) $f(x) = \dfrac{1}{x}$

  (c) $f(x) = \sqrt{x}$

  (d) $f(x) = x^5(e^x + x + 1)$

  (e) $f(x) = \dfrac{e^x - 1}{x^2}$

- **Exam 1, Problem 6**: Use the definition of the derivative to show that

$$\frac{d}{dx}x^5 = 5x^4.$$

Be sure to use connectives like equals signs and arrows as appropriate and work out any expansion that you might need completely with Pascal's triangle.

- **Exam 1, Problem 8**: The complete graph of a function f is shown in figure 1. Sketch the graph of f' on the axes provided below the graph of f.

Exam 2

- **Exam 2, Problem 2**: Differentiate the following functions:

  a. $f(x) = 3x^3 - x^2 - e^x + \cos(x)$
  b. $f(x) = (x^3 + 1)\cos(x)$
  c. $f(x) = \arcsin(x^3)$
  d. $f(x) = \dfrac{x^3}{e^{x^2+1}}$
  e. $f(x) = x^2 \left(\sin(x^3)\right)^4$

- **Exam 2, Problem 6**: The graph of $f(x) = x\,e^{-3x^2}$ restricted to the interval $[0,1]$ is shown in figure 2. Find the exact *locations* (i.e., the x-coordinates) of any absolute maxima and minima of f over that interval.

- **Exam 2, Problem 7**: I need to set up a corral to enclose a total of 400 square meters. The corral should be partitioned into four pieces, as shown in figure 3. I'll use the same fencing for each portion, interior and exterior. What are the dimensions of the least expensive corral?

- **Exam 2, Problem 8**: The function $f(x) = 4x^3 - 3x^2 - 2x + 1$ has exactly one inflection point, as shown in figure 4. Find the exact x-coordinate of that point.

Exam 3

- **Exam 3 Review, Problem 1**: In this problem, you're going to find a good rational approximation to $\sqrt{3}$. To do so, let $f(x) = x^2 - 3$. Then,

  a. Find the associated Newton's method iteration function $N(x)$.
  b. Take two Newton steps from the value $x_0 = 1$.

- **Exam 3 Review, Problem 2**: The graph of $f(x) = \frac{1}{6}x^2 + \sin(2x)$ is shown in figure Figure 5. Suppose we apply Newton's method to f starting at the initial point $x_0 = 1$.

- **Exam 3, Problem 3**: Figure 6 shows the graph of a function f over the interval $[-1,2]$. In consists of two straight line segments and a quarter-circle. Compute

$$\int_{-1}^{2} f(x)\,dx.$$

- **Exam Review 3, Problem 7**: Let $f(x) = \sin(x^2)$.

  a. Write down the right Riemann sum of f with $n = 1000$ terms over the interval $[0, 1]$.
  b. Is your estimate an overestimate or an under estimate. Why?

  You might consider the same for a left Riemann sum.

- **Exam 3, Problem 6:** Compute the values of the following definite and indefinite integrals

  a. $\int (4x^3 - 2x + 1)\, dx$
  b. $\int (e^x - \cos(x) - 1/x^2)\, dx$
  c. $\int_0^1 (x^2 - 1)\, dx$
  d. $\int_0^\pi \sin(x)\, dx$
     Please note that I do expect you to know and use the values of the trig functions at the integer multiples of π and $\pi/2$.
  e. $\int x^{10}(x^2 - 1)^2)\, dx$

- **Exam 3, Problem 7:** Use u-substitution to evaluate the following indefinite integrals:

  a. $\int x\, e^{-x^2}\, dx$
  b. $\int x^3(x^2 - 1)^{10}\, dx$
     Note that this is a *corrected* version of the bogus exam question.

New material

1. Use u-substitution to evaluate the following definite integrals. Be sure to change the bounds of integration as appropriate!

   a. $\int_0^2 x^2(x^3 + 1)^{10} dx,$
   b. $\int_0^{\sqrt{\pi}} x \sin(x^2).$

2. Use u-substitution to translate the following normal integrals into *standard* normal integrals

   a. $\frac{1}{\sqrt{2\pi}\,2} \int_2^{10} e^{-(x-4)^2/8}\, dx$
   b. $\frac{1}{\sqrt{2\pi}\,3} \int_{-4}^{5} e^{-(x+1)^2/18}\, dx$

3. Figure 7 shows an empty set of axes. Draw on that set of axes two normal distributions

   - The normal distribution with mean -2 and standard deviation 1 and
   - The normal distribution with mean 3 and standard deviation $1/2$.

Be sure to pay special attention to both the *placement* of the mean and the *relative concentration* about that mean.

Figures

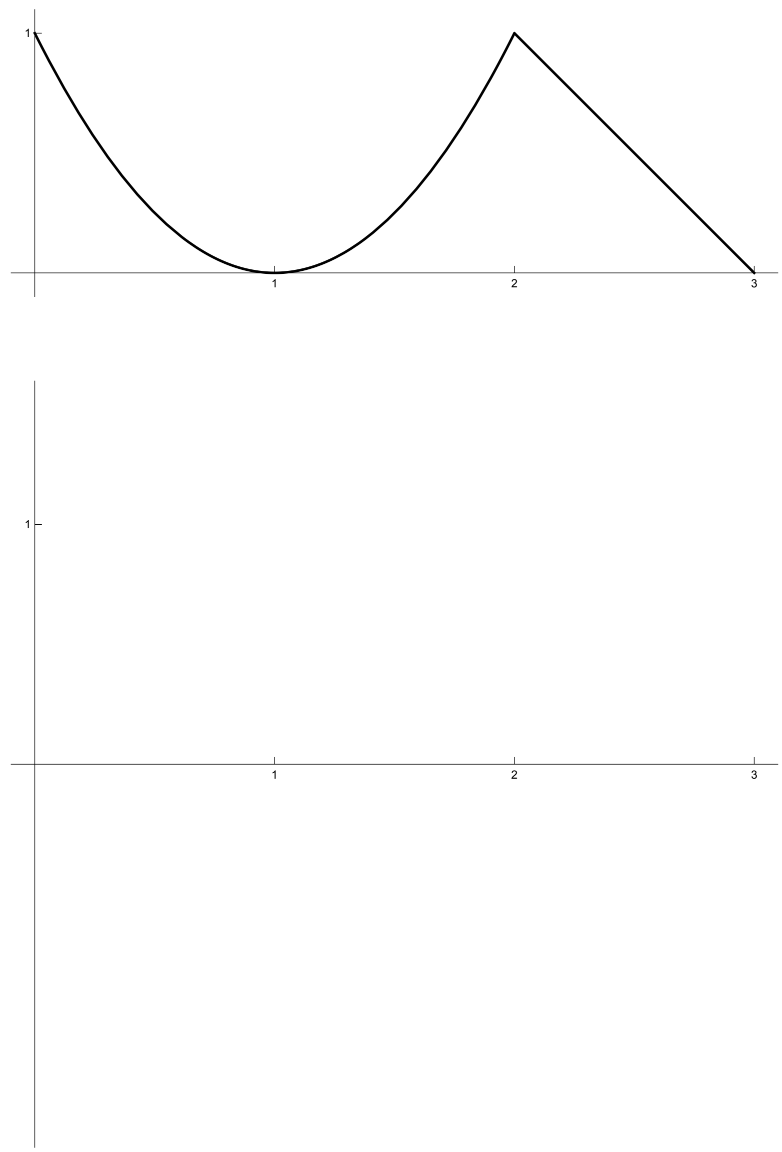

Figure 1: Draw the derivative

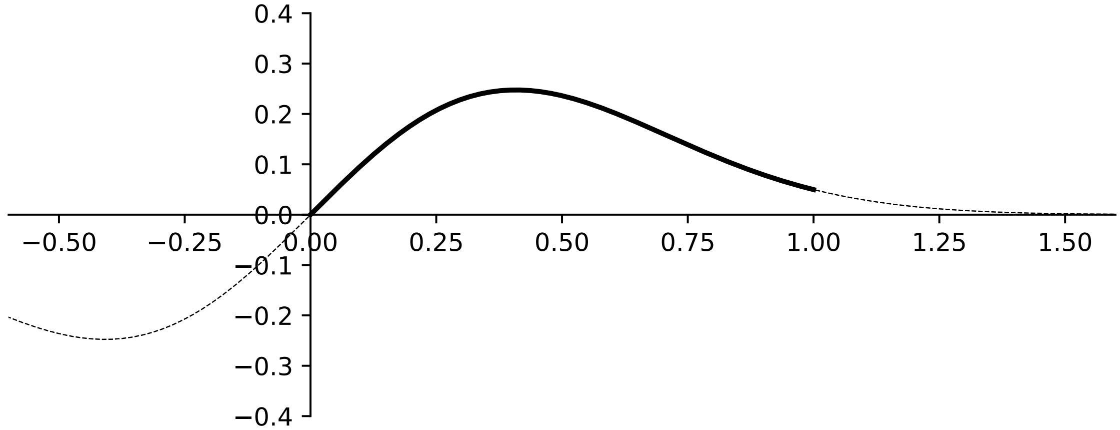

Figure 2: Plot of a function

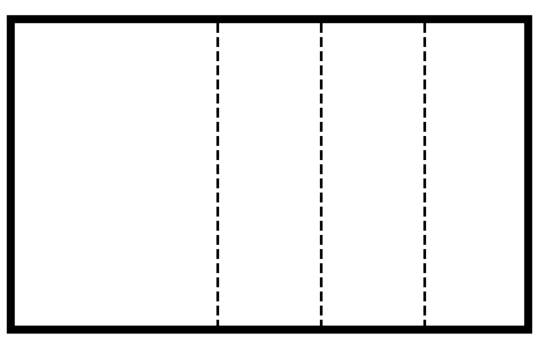

Figure 3: A divided corral

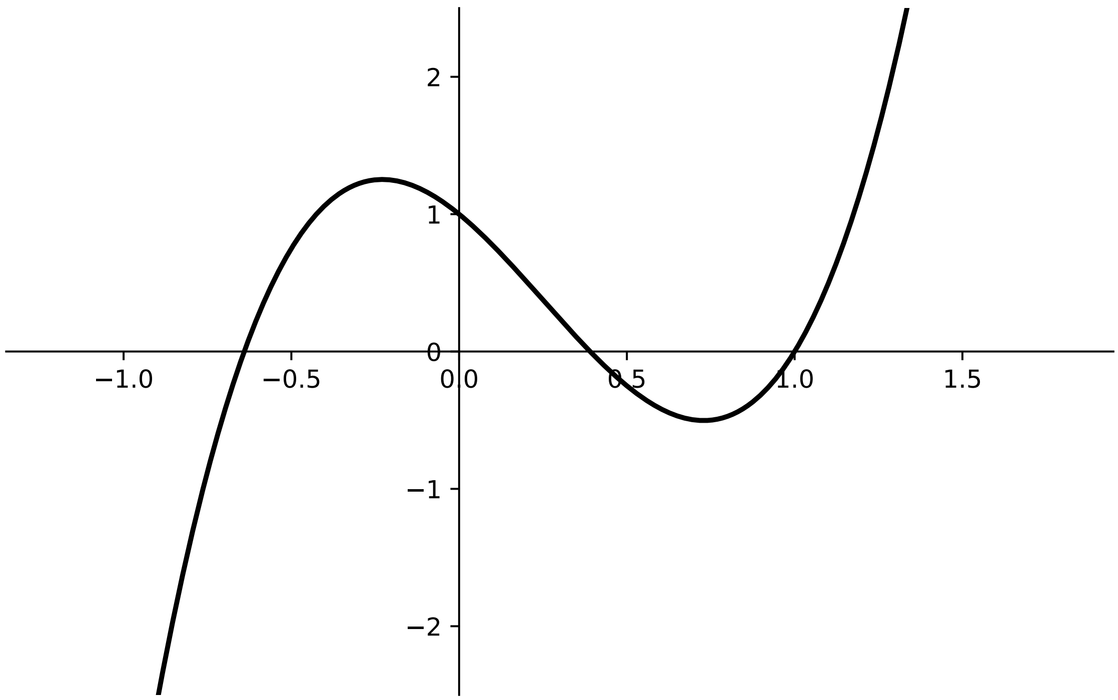

Figure 4: The graph of $y = 4x^3 - 3x^2 - 2x + 1$

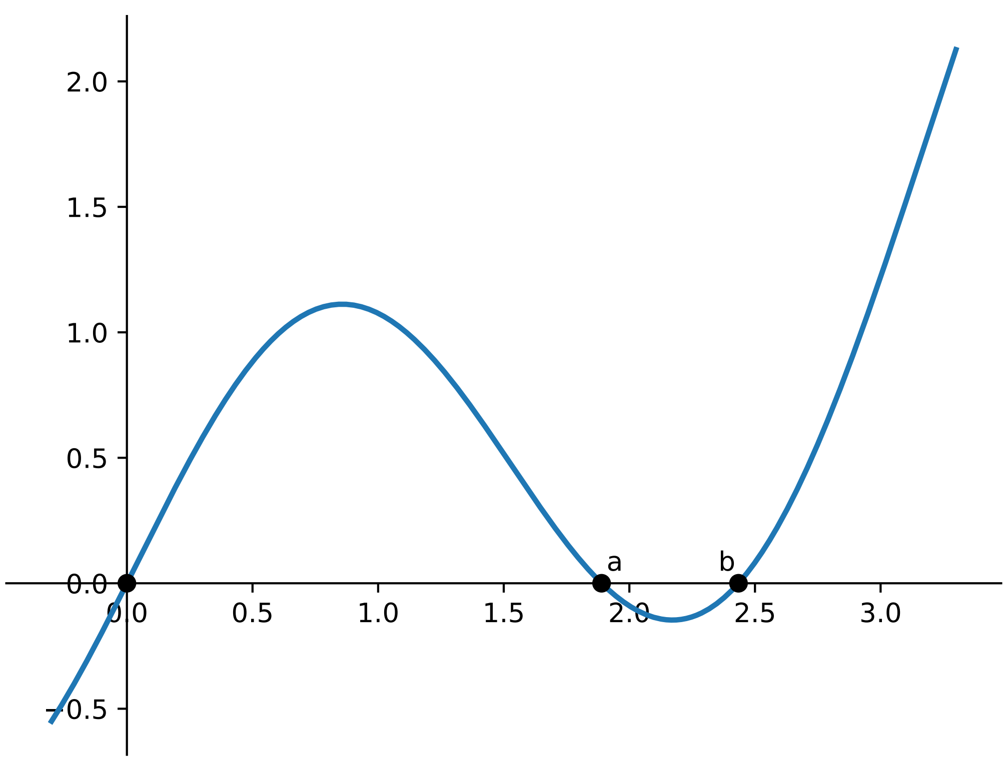

Figure 5: The graph of $f(x) = \frac{1}{6}x^2 + \sin(2x)$

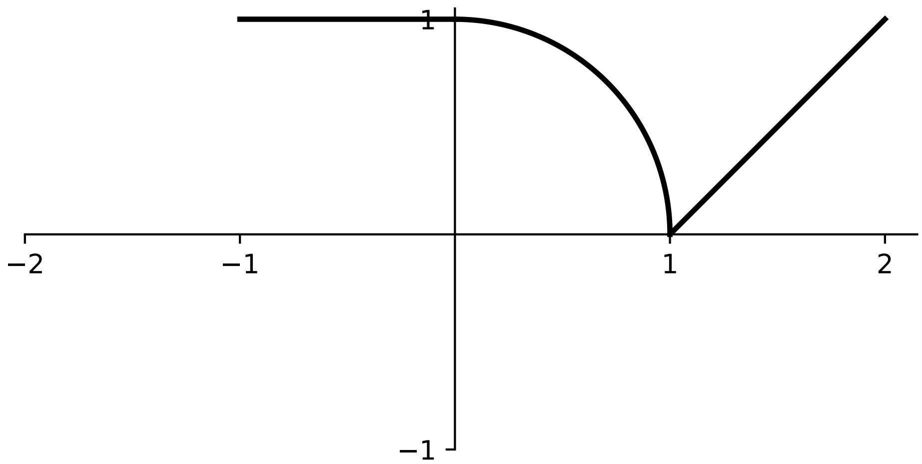

Figure 6: The graph of a function to integrate

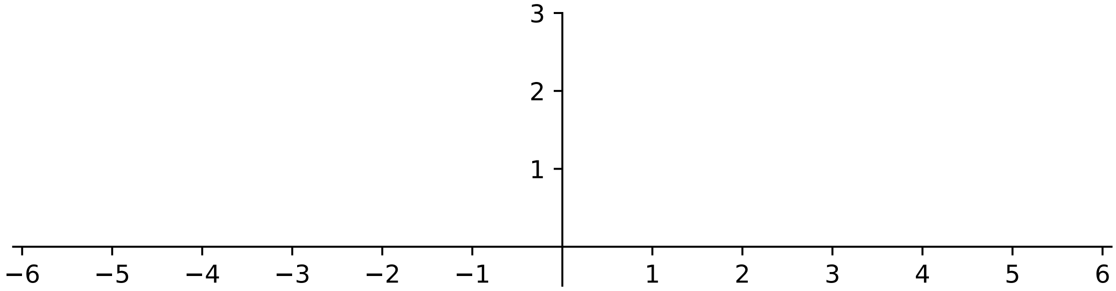

Figure 7: A pair of axes waiting for normal distributions