# Calc I - Review for Exam 3

The third exam will be this Friday, March 18. Please try to solve the problems on this review sheet to the best of your ability before class on Wednesday! All the problems on the exam will likely look like something you see on this problem sheet.

1. In this problem, you're going to find a good rational approximation to $\sqrt{3}$. To do so, let $f(x) = x^2 - 3$. Then,

   a. Find the associated Newton's method iteration function $N(x)$.
   b. Take two Newton steps from the value $x_0 = 1$.

   *Comment:* This is a *corrected* version of the problem that was on the review for quiz 3.

2. The graph of $f(x) = \frac{1}{6}x^2 + \sin(2x)$ is shown in figure Figure 1. Suppose we apply Newton's method to $f$ starting at the initial point $x_0 = 1$.

   a. Sketch the tangent line at the point $x_0 = 1$ and illustrate how $x_1$ is found.
   b. Repeat that process for $x_1$ to illustrate how $x_2$ is found.
   c. To which of the roots $x = 0$, $x = a$, or $x = b$ marked in the figure is the process most likely to converge?

   *Comment:* This link on our class webpage provides some clues for inspiration, as well as a numerical tool to check your work.

3. Suppose I throw a rock off a 30 foot high cliff with a vertical velocity of 8 feet per second.

   a. Find an equation for the height $y(t)$ as a function of time.
   b. How high does the rock go?

   Recall that the acceleration due to gravity near the surface of the earth is $-32\,\text{ft}/\text{s}^2$.

4. Suppose I'm driving down a straight road at 65 mph. A kindly police officer is 1/4 mile down the road and 1/10 mile off the road. What speed does the kindly police officer's radar detector register at that point in time?

5. Figure Figure 2 shows the graph of a function $f$ over the interval $[-2, \sqrt{2}]$. In consists of a semi-circle, a straight line segment, and $1/8^{\text{th}}$ the arc of a final circle. Compute

$$\int_{-2}^{\sqrt{2}} f(x)\,dx.$$

6. Let $f(x) = 4x^2$.

   a. Write down the left Riemann sum of $f$ with $n = 4$ terms over the interval $[0, 2]$
   b. Draw a picture to illustrate.
   c. Is your estimate an overestimate or an under estimate. Why?

   You might consider the same for a right Riemann sum.

7. Let $f(x) = \sin(x^2)$.

   a. Write down the right Riemann sum of $f$ with $n = 1000$ terms over the interval $[0, 1]$.
   b. Is your estimate an overestimate or an under estimate. Why?

   You might consider the same for a left Riemann sum.

8. In this problem, you're going to show that

$$\int_0^1 x^5 \, dx = \frac{1}{6}$$

   directly from the definition of the Riemann integral. Thus, you should

   a. Express the integral as a Riemann sum,
   b. Manipulate the Riemann sum to isolate $\sum_{i=1}^{n} i^5$, and
   c. Compute the limit as $n \to \infty$.
      You might could use the fact that

$$\sum_{i=1}^{n} i^5 = \frac{n^2(n+1)^2\left(2n^2 + 2n - 1\right)}{12}.$$

9. Compute the values of the following definite and indefinite integrals

   a. $\displaystyle\int_0^2 x^3 \, dx$
   b. $\displaystyle\int x^3(2x + 5) \, dx$
   c. $\displaystyle\int_0^\pi \sin(x) \, dx$
   d. $\displaystyle\int (e^x - \sin(x)) \, dx$
   e. $\displaystyle\int_0^2 \left(8x^2 + x + 1\right) \, dx$
   f. $\displaystyle\int_{-2}^2 x^2 \sin(x^{99}) \, dx$

10. Use $u$-substitution to evaluate the following indefinite integrals:

a. $\int x \cdot \cos(x^2)\, dx$

b. $\int \frac{1}{(3x+1)^2}\, dx$

## Figures

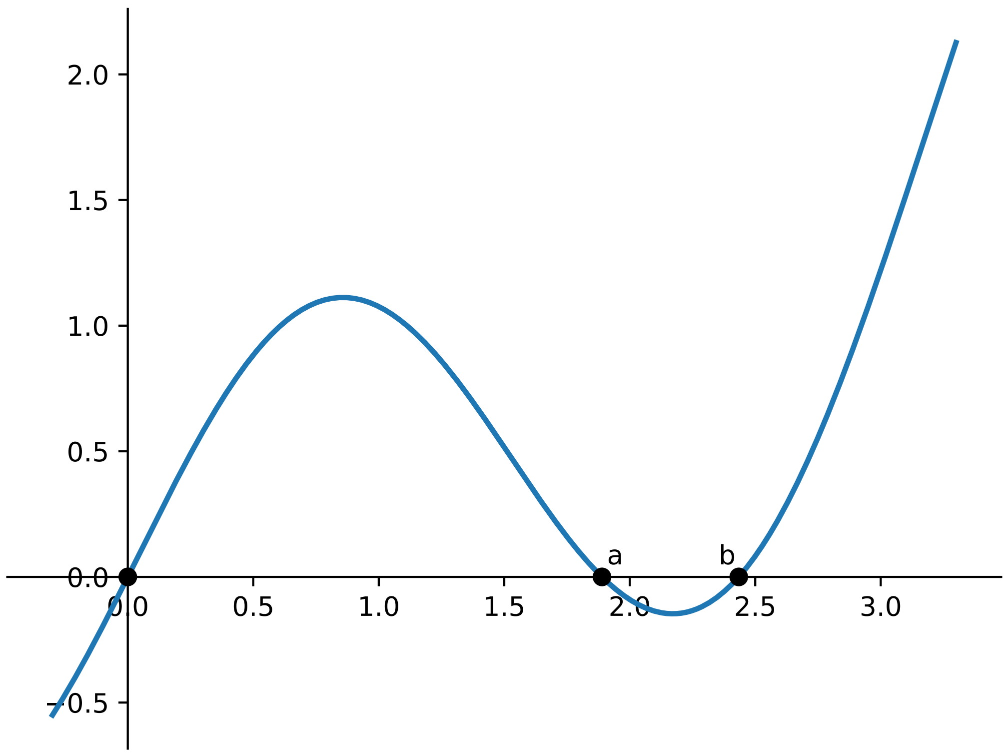


Figure 1: The graph of $f(x) = \frac{1}{6}x^2 + \sin(2x)$

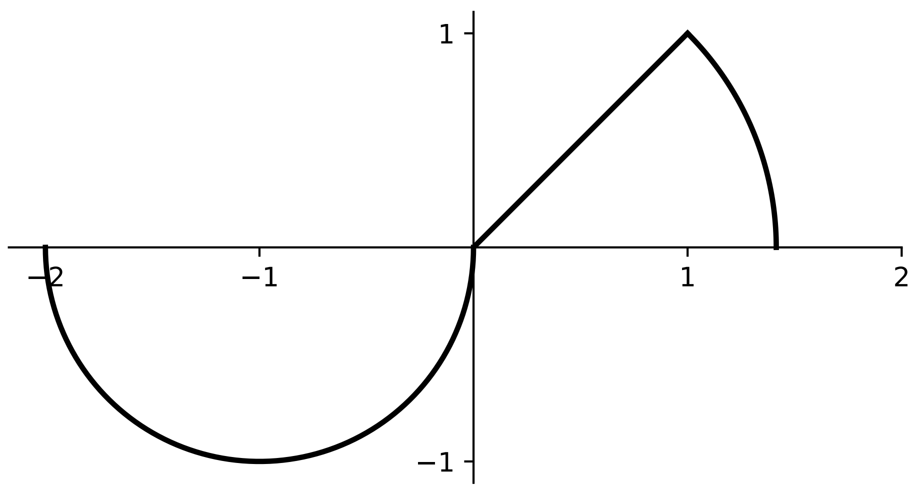


Figure 2: The graph of a function to integrate