Calc I - Review for Exam 2

The second exam will be this Friday, March 7. Please try to solve the problems on this review sheet to the best of your ability before class on Wednesday! All the problems on the exam will likely look like something you see on this problem sheet.

Differentiation Problems

1. Differentiate the following functions:

   a. $f(x) = 3x^5 - 7x^3 + 4x - 2$

   b. $g(x) = e^x + \ln(x)$

   c. $h(x) = \sin(x) + \cos(x) - \tan(x)$

   d. $p(x) = \arcsin(x) + \arctan(x^3)$

   e. $q(x) = \dfrac{x^2+1}{x-3}$

   f. $r(x) = \sin(x^3 + x)$

   g. $s(x) = (x^2+1)e^x$

   h. $t(x) = \ln(\cos x)$

   i. $u(x) = \dfrac{x^3}{e^{x^2+1}}$

   j. $v(x) = x^2 \left(\arcsin(3x^2)\right)^5$

2. Compute the following limits:

   a. $\displaystyle\lim_{\theta\to 0} \frac{\sin(3\theta)}{\theta}$

   b. $\displaystyle\lim_{\theta\to 0} \frac{\tan(2\theta)}{\theta}$

   c. $\displaystyle\lim_{\theta\to 0} \frac{1-\cos(\theta)}{\theta}$

   Comment: You may assume the fact that $\displaystyle\lim_{\theta\to 0} \frac{\sin(\theta)}{\theta} = 1$.

3. In this problem, we'd like to take a step towards understanding why

$$\frac{d}{dx}\sin(x) = \cos(x).$$

To do so, use the following outline:

a. Write down the difference quotient for the sine function,
b. Apply the sine summation formula to expand $\sin(x+h)$,
c. Do a little algebra to separate the $\sin(x)$ and $\cos(x)$ that you see in the last step, and
d. Identify the expressions whose limits you need to finish the job and what their limiting values should be.

Comments:

- After step (c), you should have $\sin(x)$ and $\cos(x)$ multiplied by fractions with an h in the denominator.
- In part (d), you need to identify what the limits of those fractions need to be in order for things to work; you *don't* need to go through all the work of proving what those limitig values are.
- In part (b), you'll need to use the sine summation formula
$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$.

4. Use the quotient rule to establish the following differentation rules:

a) $\dfrac{d}{dx}\tan(x) = \sec^2(x)$
b) $\dfrac{d}{dx}\sec(x) = \sec(x)\tan(x)$

5. Differentiate the following functions using logarithmic differentiation:

a. $f(x) = (x^2+1)^5\dfrac{\sqrt{x}}{e^x}$
b. $g(x) = \dfrac{(x^3+2x+1)^4}{\sin^2(x)\ln(x)}$
c. $h(x) = x^{2x}$

6. Figure 1 shows the graph of $f(x) = x^3 + \frac{1}{2}x - \frac{3}{2}$ together with the dashed line $y = x$.

a. Sketch the graph of f^{-1} on the same set of axes.
b. Find an equation for the line that is tangent to the graph of f^{-1} at the point where $x = 0$.

7. Let $f(x) = x^3 - 12x + 12$ restricted to the interval $[0,4]$. Find the absolute maximum and minimum values of f over that interval.

8. I need to set up a corral to enclose a total of 1000 square meters. The corall should be partitioned into three pieces, as shown in figure 2. I'll use the same fencing for each portion, interior and exterior. What are the dimensions of the least expensive corall?

9. Suppose I inscribe a rectangle under the graph of $y = e^{-x^2}$ with one vertex at the origin, as shown in figure 3. What are the dimensions and area of the largest such rectangle?

10. The function $f(x) = 4x^3 + 3x^2 + 2x + 1$ has exactly one inflection point, as shown in figure 4. Find the exact x-coordinate of that point.

Figures

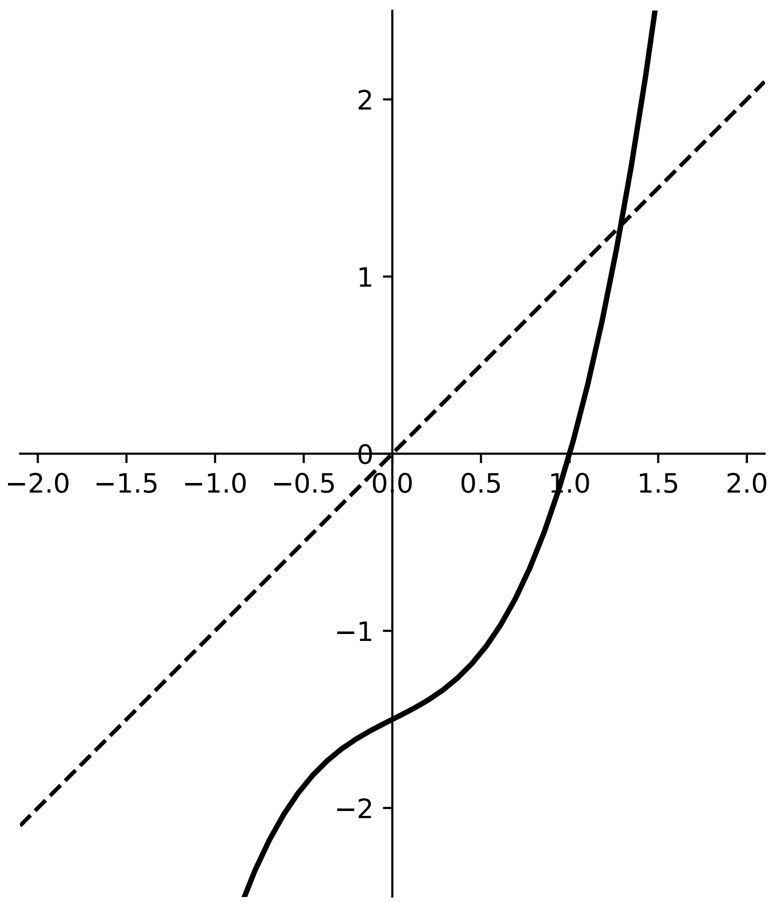

Figure 1: Plot of a $1-1$ function

Figure 2: A divided corral

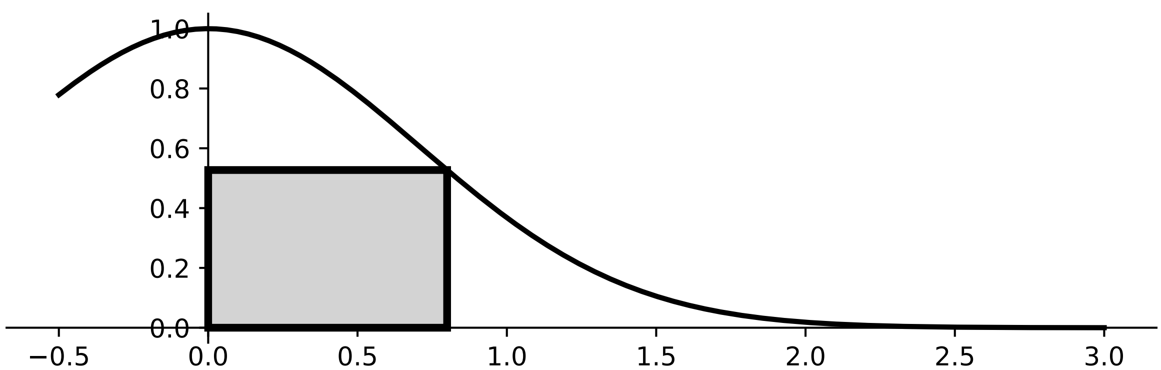

Figure 3: A rectangle inscribed under the graph of $y = e^{-x^2}$

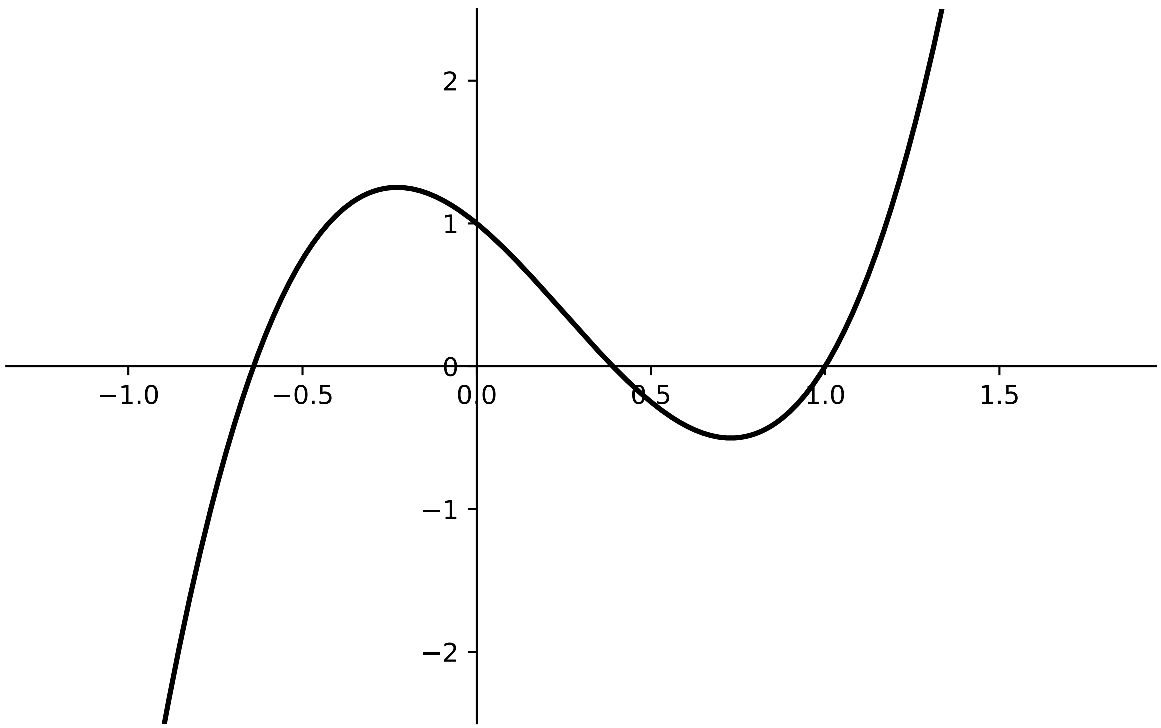

Figure 4: The graph of $y = 4x^3 - 3x^2 - 2x + 1$