Calc I - Review for Exam 1

The first exam will be this Friday, February 7. Please try to solve the problems on this review sheet to the best of your ability before class on Wednesday! All the problems on the exam will likely look like something you see on this problem sheet.

1. Use the following steps to *estimate* the derivative of $f(x) = 6^x$.

   a) Write down the difference quotient for f.
   b) Use a little algebra to separate the xs from the hs.
   c) Refer to the following table to help you find your final estimate.

h	0.1	0.01	0.001	0.0001	0.00001	0.000001
$\dfrac{6^h - 1}{h}$	1.96231	1.80791	1.79337	1.79192	1.79178	1.79176

2. Let $f(x) = x^2 - x$.

   (a) Draw the graph of f.
   (b) Write down the difference quotient for f and simplify it to the point where you can cancel the h in the denominator.
   (c) Write down the estimate for the slope of the graph of f at $x = 1$ yielded by the difference quotient using $h = 0.2$.
   (d) Write down an equation for the corresponding secant line through $(1, f(1))$ and $(1.2, f(1.2))$ and draw it on your graph.
   (e) Write down and equation for the precise tangent line at $x = 1$ and draw that on your graph as well.

3. Figure 1 shows the complete graph of a function f; its domain is $(-1, 2]$.

   (a) What are $f(1.5)$ and $\lim_{x \to 1.5} f(x)$?
   (b) What are $\lim_{x \to 1^-} f(x)$, $\lim_{x \to 1^+} f(x)$, and $\lim_{x \to 1} f(x)$?
   (c) What can you say about $\lim_{x \to -1^+} f(x)$.

4. Compute the following limits.

1. $\lim\limits_{x \to 2} \dfrac{3x^2 - 7x + 2}{x - 2}$

2. $\lim\limits_{x \to 3^+} \dfrac{3x^2 - 7x + 2}{x - 3}$

5. Write down a complete sentence referring to the intermediate value theorem explaining why $f(x) = 3x^7 - x - 1$ has a root between $x = 0$ and $x = 1$.

6. Find the derivatives of the following functions, *using the definition of the derivative.*

   (a) $f(x) = 2x^2 - 4x$
   (b) $f(x) = 1/\sqrt{x}$
   (c) $f(x) = x^6$

7. Find the derivatives of the following functions, using any technique you see fit.

   (a) $f(x) = 2x^2 - 4x$

   (b) $f(x) = 1/\sqrt{x}$

   (c) $f(x) = x^6$

   (d) $f(x) = x^5 \left(x^2 - x - 1 \right)$

   (e) $f(x) = (x^5 + x^3 + x)(x^6 - x^4 + x^2 - 1)$

   (f) $f(x) = \left(x^2 - x - 1 \right) / x$

   (g) $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$

   (h) $f(x) = e^x + x^e$

   (i) $f(x) = e^x (x^3 - x - 1)$

8. The complete graph of a function f is shown in Figure 2; it consists of a line segment, a quarter-circle, and a semi-circle. Sketch the graph of f'.

9. Use the defintion of the derivative to prove each of the following differentation rules:

   a) $\dfrac{d}{dx}(2f(x) + 3g(x)) = 2f'(x) + 3g'(x)$

   b) $\dfrac{d}{dx}\left(x^2 f(x) \right) = 2x f(x) + x^2 f'(x)$
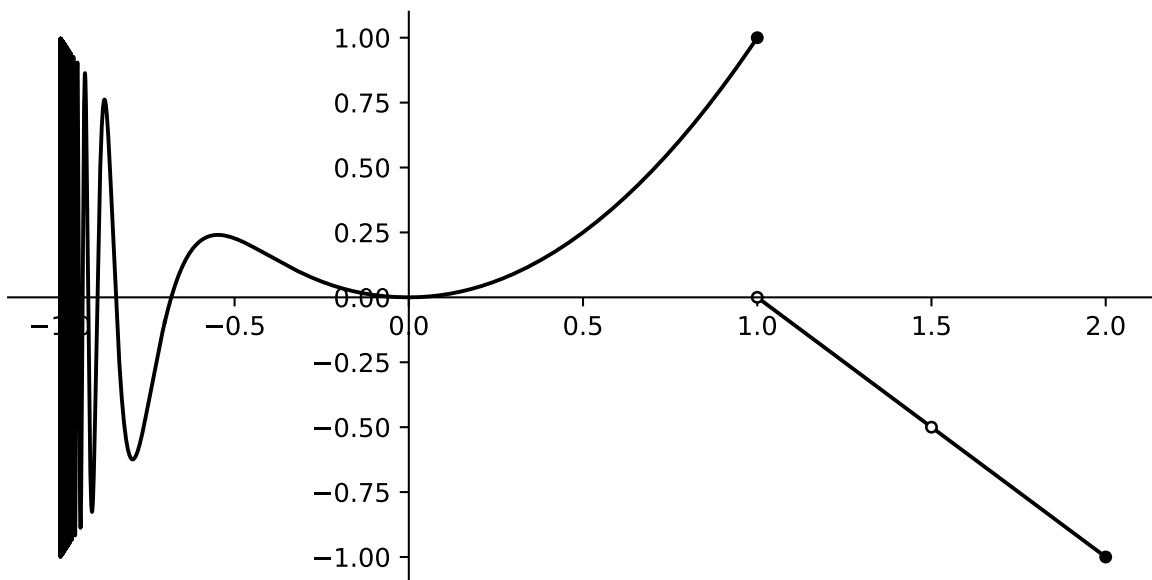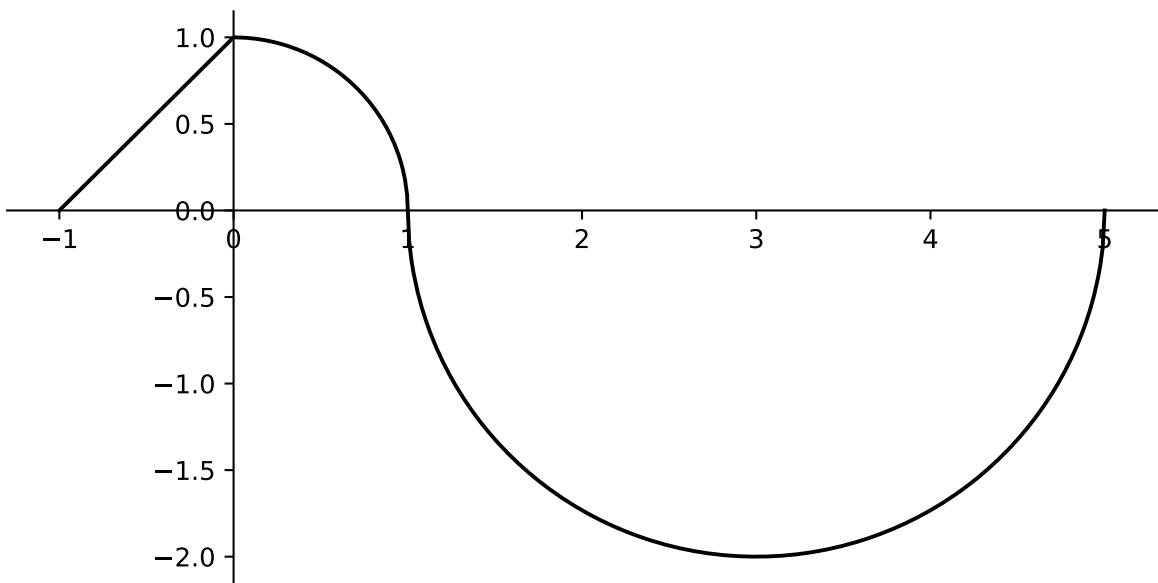
Figures

Figure 1: Figure for limits and continuity

Figure 2: Figure for a derivative sketch