Calc II - Review for exam III

The third exam will be this Friday, April 26. We will discuss some of these problems in class on Wednesday, but you should work them all out to the best of your ability prior to that. Understanding the problems on this sheet will help you greatly on the exam.

1. Use the integral test to show that $\sum_{n=3}^{\infty} \frac{1}{n(\ln(n))^p}$ converges precisely when $p > 1$.

2. Suppose we'd like to approximate
$$\sum_{n=1}^{\infty} \frac{n}{n^4+1}$$
by truncating the sum to obtain a finite sum of the form
$$\sum_{n=1}^{N} \frac{n}{n^4+1}.$$
How large does N have to be to ensure that our approximation is within 0.0001 of the actual value?

3. Write down a couple of complete sentences using the comparison test to show that
$$\sum_{n=1}^{\infty} \frac{\sin\left(n^3\right)}{n^4}$$
converges absolutely.

4. Write down a couple complete sentences using the alternating series test to show that
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln(n)}{n}$$
converges conditionally.

5. Classify the following series as absolutely convergent, conditionally convergent, or divergent.
   (a) $\sum (-1)^n \frac{n^2}{n^2+2}$
   (b) $\sum (-1)^n \frac{n^2}{n^3+2}$
   (c) $\sum (-1)^n \frac{n^2}{n^4+2}$

6. Find the domain of convergence of the following power series.
   (a) $\sum (-1)^n \frac{n}{5^n} x^n$
   (b) $\sum (-1)^n \frac{1}{n5^n} x^n$

7. Find the radius of convergence of $\sum (-1)^n \frac{n!}{n^n} x^n$.

8. Starting with the geometric series formula, find a power series for $x/(1+x^7)$, then find a series representation of
$$\int \frac{x}{1+x^7} dx.$$

9. Let's use Taylor's formula to find the power series expansion of $f(x) = \sin(2x)$. To do so, we'll use the following outline:
   (a) Find the first 4 or 5 derivatives of f,
   (b) Evaluate those derivatives of $x = 0$,
   (c) Note the patterns in the values,
   (d) Conjecture a formula for $f^{(2n+1)}(0)$
   (e) Use your formula to write down the power series.
10. Find the domain of convergence of

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n\,2^n}(x-3)^n.$$