Final review problems off of old exams

We have a final exam this coming Friday, May 3 in the usual classroom. Note that the 8:00 AM section will meet at 8:00 as usual but the 11:00 AM section will start at 11:30. Here are some problems to help focus your study. These are mostly off of old exams but there are a few variations off of the problems from Exam III.

Problems from Exam I

1. Evaluate the following integrals.

   a) $\displaystyle\int_0^2 x\sqrt{4-x^2}\,dx$

   b) $\displaystyle\int x\sin(x)dx$

   c) $\displaystyle\int \cos^3(x)\,dx$

   d) $\displaystyle\int \frac{8x-1}{(x-2)(2x+1)}\,dx$

   e) $\displaystyle\int_0^2 x^2\sqrt{4-x^2}\,dx$

2. The complete graph of a function f is shown in Figure Figure 1; it consists of three straight line segments. Evaluate $\displaystyle\int_{-1}^4 f(x)\,dx$.

3. Suppose we wish to estimate

$$\int_0^2 (-\sin(x^2/3))\,dx$$

   with a midpoint sum and we'd like our result to be within 0.0001 of the actual value.

a) Find an n large enough so that n terms will guarantee your estimate is within the desired accuracy.

b) Write down the resulting sum using summation notation.

Note that the graph of the integrand $f(x) = -\sin(x^2/3)$, together with its first and second derivatives, is shown in Figure Figure 2

4. Evaluate the following improper integral:

$$\int_1^\infty \frac{3}{x^5}\,dx.$$

5. The function graphed in Figure Figure 3. is $f(x) = 1 - x^2$. Suppose we spin the shaded region around the x-axis. Express the volume of the resulting solid as an integral.

   Also: It might be worth thinking about what would happen if you spin one half period of the sine function around the x-axis. The region I'm thinking of is shown in Figure Figure 4.

Problems off of Exam II

1. Use u-substitution to translate the following normal integral into a standard normal integral:

$$\frac{1}{\sqrt{2\pi}\,5}\int_5^{20} e^{-(x-10)^2/50}dx.$$

5. Use the limit laws for sequences to explain clearly why

$$\lim_{n\to\infty}\frac{4n+1}{2n-2} = 2.$$

6. Suppose that $a_0 = 0$ and $a_{n+1} = a_n^2 - 1$.

   a) What is the value of a_4?

   b) What is the value of a_{100}?

7. Evaluate the sum

$$\sum_{n=1}^{\infty}\frac{2}{n(n+1)}$$

   by

   - Decomposing the term $2/(n(n+1))$ into partial fractions and then
   - Use the fact that the result collapses.

8. Use the geometric series formula to express

$$\sum_{n=3}^{\infty}(-1)^{n+1}\frac{2^n}{5^{n-2}}$$

as a simple, finite combination of fractions.

Problems from Exam III

1. Use the integral test to show that

$$\sum_{n=1}^{\infty}\frac{1}{n^p}$$

converges precisely when $p > 1$.
Note: You can't assume anything about p-series, since we're trying to prove a basic fact about such series. You can feel free to assume this basic fact for the rest of the exam, though!
Also note: Make sure you can evaluate improper p-integrals directly. that is, you should absolutely be able to evaluate

$$\int_1^{\infty}\frac{1}{x^p}\,dx.$$

2. Suppose we'd like to approximate

$$\sum_{n=1}^{\infty}(-1)^n\frac{n}{n^4+1}$$

by truncating the sum to obtain a finite sum of the form

$$\sum_{n=1}^{N}(-1)^n\frac{n}{n^4+1}.$$

Find a lower bound on N that will ensure that our approximation is within 0.001 of the actual value.
Also, if I asked you how to estimate either of the following to within 0.00001, you should be able to tell if you should use a the estimate arising from either the alternating series test or the integral test:

- $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^3}$
- $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{1}{n^3}$

3. Write down a couple of complete sentences using the comparison test to show that

$$\sum_{n=1}^{\infty} \frac{\cos\left(n^9\right)}{n^3}$$

converges absolutely.
Also, Write down a couple of complete sentences using the alternating series test to show that

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$$

converges conditionally.

4. Starting with the geometric series formula, find a power series for $2x/(1+8x^3)$, then find a series representation of

$$\int \frac{2x}{1+8x^3} dx.$$

Also: Starting with the power series representation of the exponential function, find a power series representation of

$$\int e^{-x^2}\, dx.$$

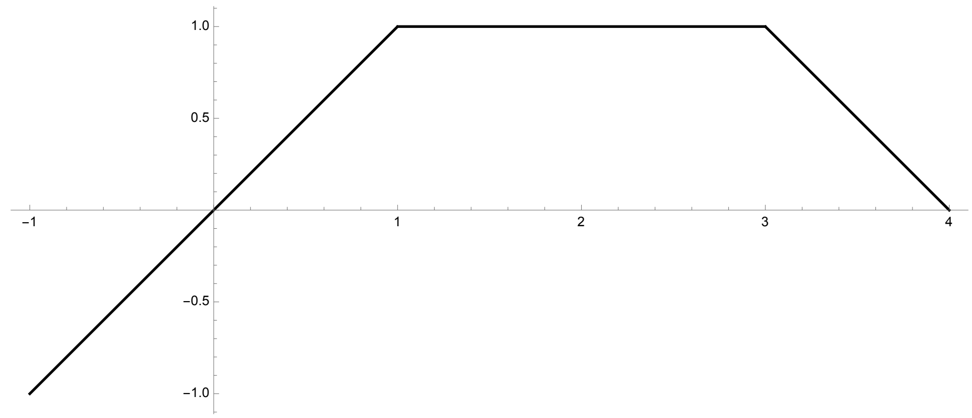

Figure 1: A piecewise function

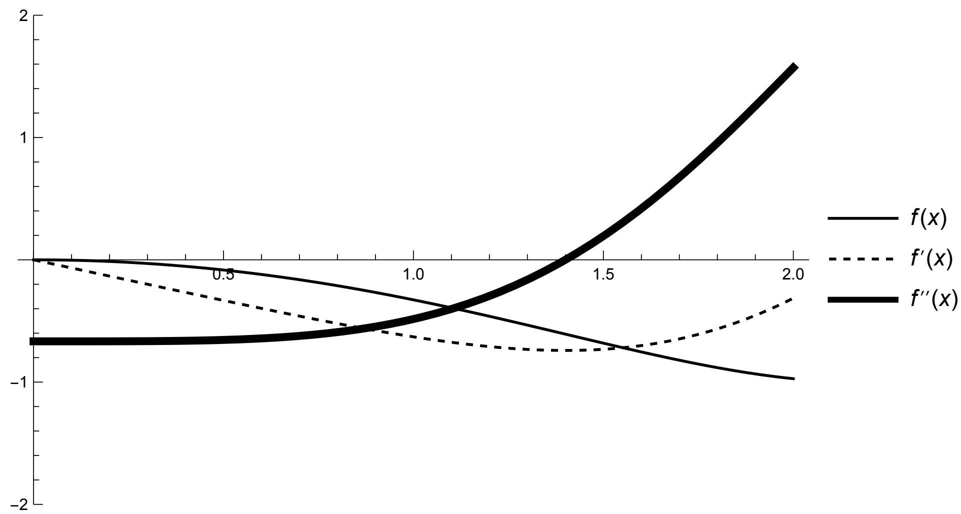

Figure 2: The graphs of f, f', and f''

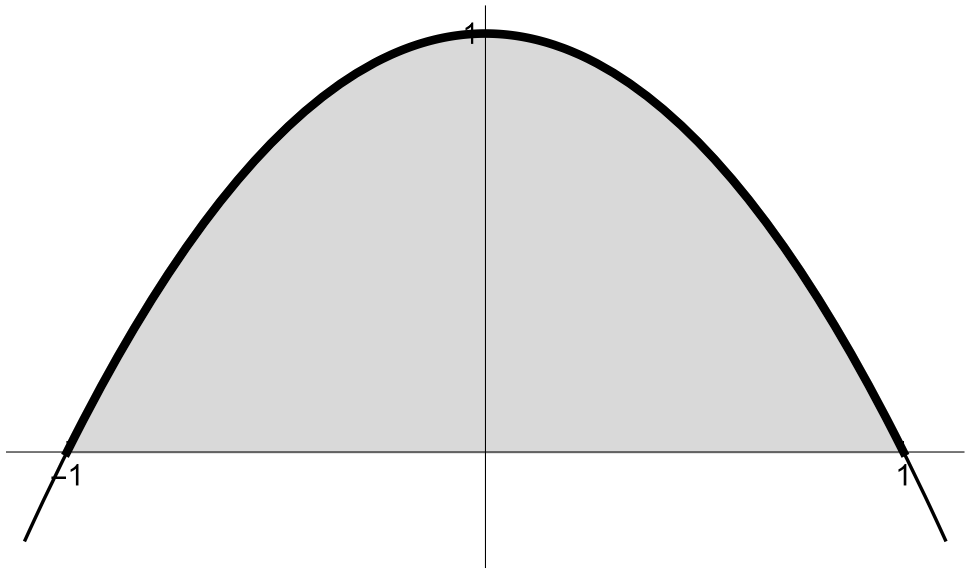

Figure 3: The graph of $1 - x^2$

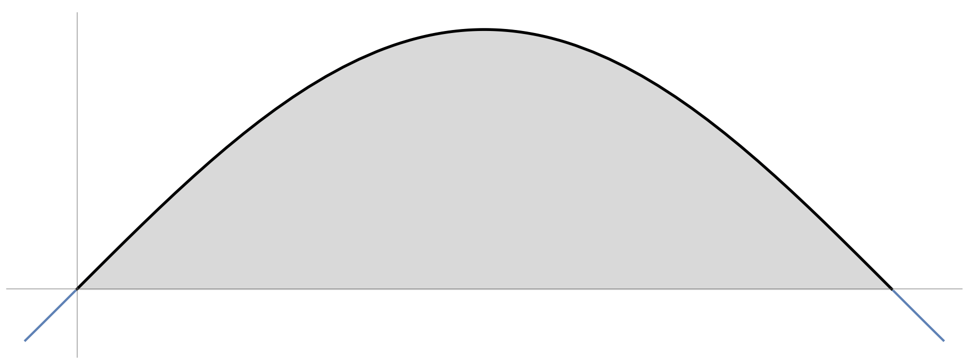

Figure 4: The graph of the sine function