

## Calc III - Review 2

- Let  $f(x, y) = xy^3$ .
  - Compute  $\nabla f$ .
  - From the point  $(2, 1)$ , in what direction  $\vec{u}$  is  $f$  changing the fastest?
  - From the point  $(2, 1)$ , is there any direction  $\vec{u}$  so that  $D_{\vec{u}}f(2, 1) = 10$ ?
- Find all points on the curve  $x^2 + y^2 - xy = 4$  where a normal vector to the curve is perpendicular to the vector  $\langle 1, 2 \rangle$ .
- Find and classify the critical points of the function  $f(x, y) = 8x^2 + 2xy^2 + 4y^2$ .
- Find the equation of the plane tangent to the graph of  $2x^2 - y^2 + z^2 = 8$  at the point  $(2, 1, 1)$ .
- Let  $f(x, y) = 3(x - 2)^2 + (y + 1)^2$ .
  - Sketch and label several contours of  $f$ .
  - What does part (a) tell you about local maximum and minimum values of  $f$ ?
  - Sketch the gradient field of  $f$  over your contour diagram. Pay special attention to the direction and relative magnitudes of the gradient vectors.
- Use the method of Lagrange multipliers to find the extremes of  $f(x, y) = 2x + 4y$  subject to  $x^2 + y^2 = 20$ .
- In this problem, we're going to take the first step toward finding the regression line for the points

$$\{(-2, 1), (0, 0), (1, 1)\}.$$

To do so,

- Write down the algebraic formula expressing the least squared errors that you need to minimize. Your formula should involve two parameters representing the coefficients in the formula for a line.
  - Using partial derivatives, write down the resulting linear equations that you need to solve to find the coefficients.
- Figure 1 (a) shows the contour diagram of  $f(x, y) = xye^{-(x^2+y^2)}$ .
    - On the figure, identify any maximum, minimum, or saddle points.
    - Find the exact location of all maxima.
  - The contour plot of a function is shown in figure 2.
    - For each point that I've marked on the figure, draw the corresponding gradient vector emanating from that point. Be sure to pay special attention to both the direction and relative magnitude of your gradient vectors.
    - Label any max, min, or saddle points that you see.
  - Evaluate

$$\int_0^3 \int_0^2 (3x^2 + 4y^3) dx.$$

11. Let  $R$  denote the region bound between the graphs of

$$y = x^2 - x - 4 \text{ and } y = x + 4.$$

Set up

$$\iint_R f(x) dA$$

as an iterated integral.

12. Figure 3 shows a contour plot of a function  $f$  over the square  $S = [0, 2] \times [0, 2]$ . Write down two Riemann sums to estimate

$$\iint_S f(x) dA.$$

One sum should be a good upper bound and the other a good lower bound.

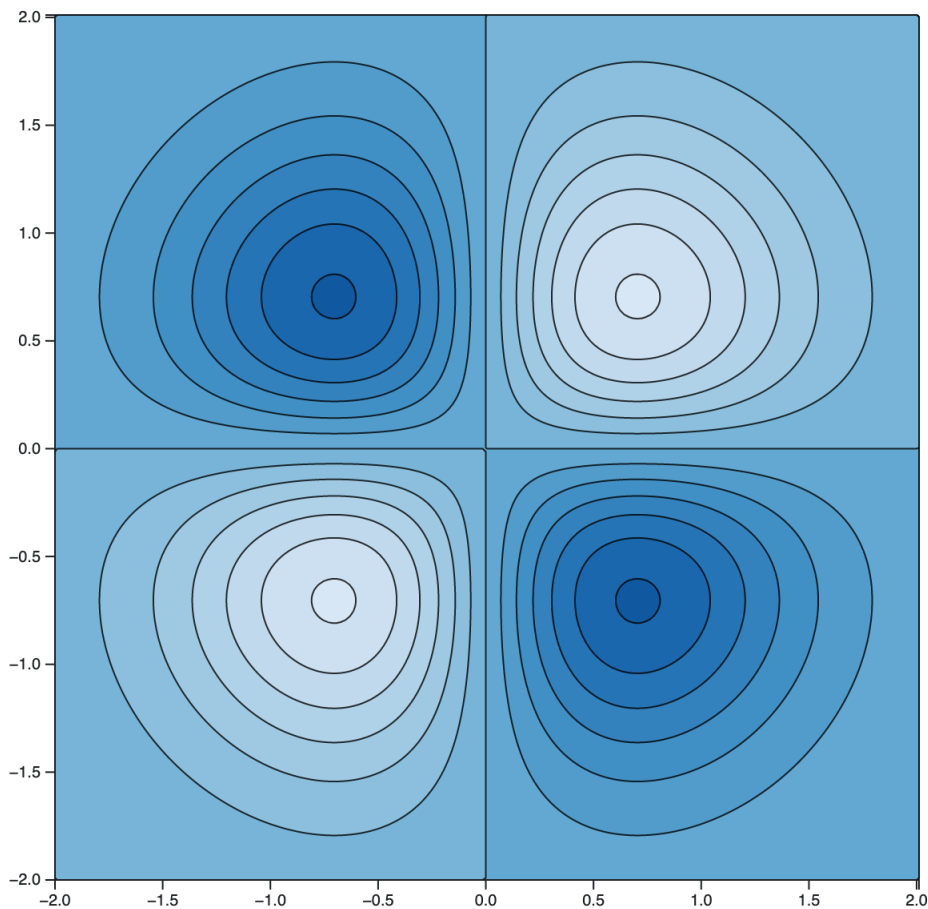


Figure 1: The contour diagram of  $f(x, y) = xye^{-(x^2+y^2)}$ .

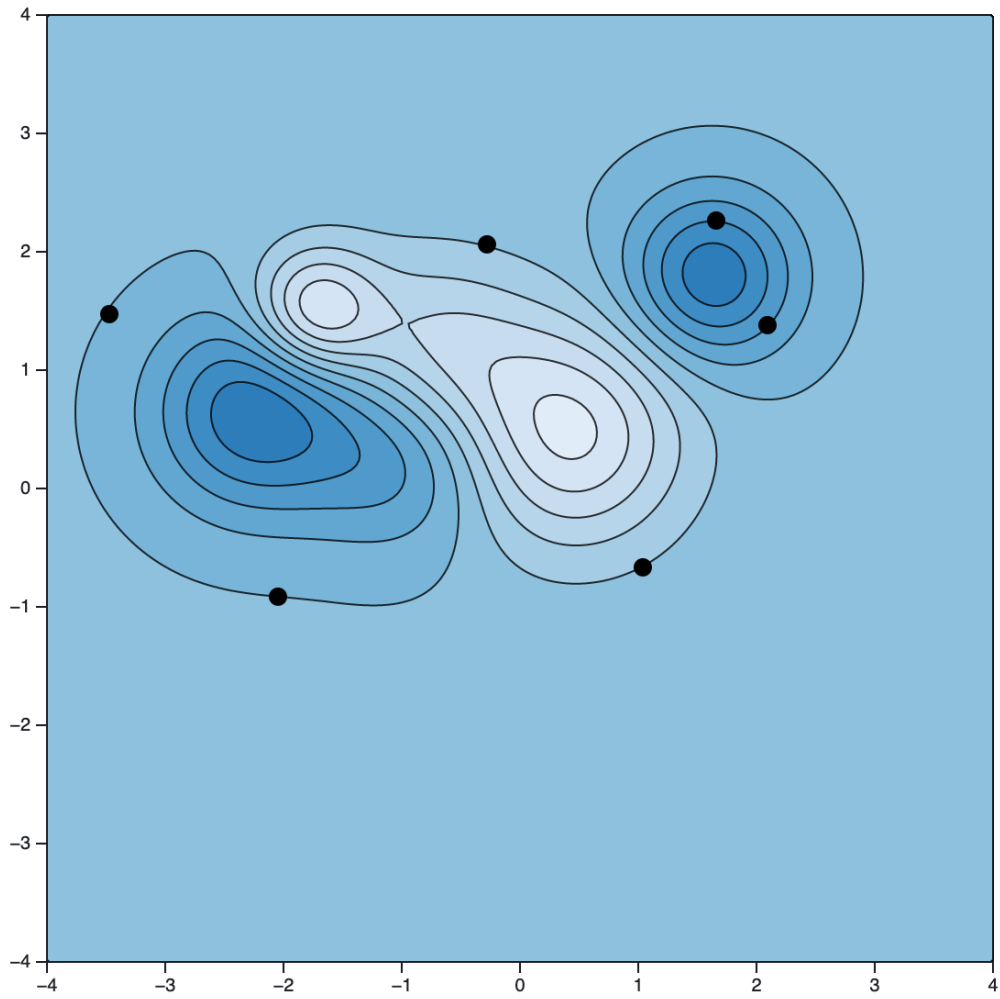


Figure 2: A contour plot with points

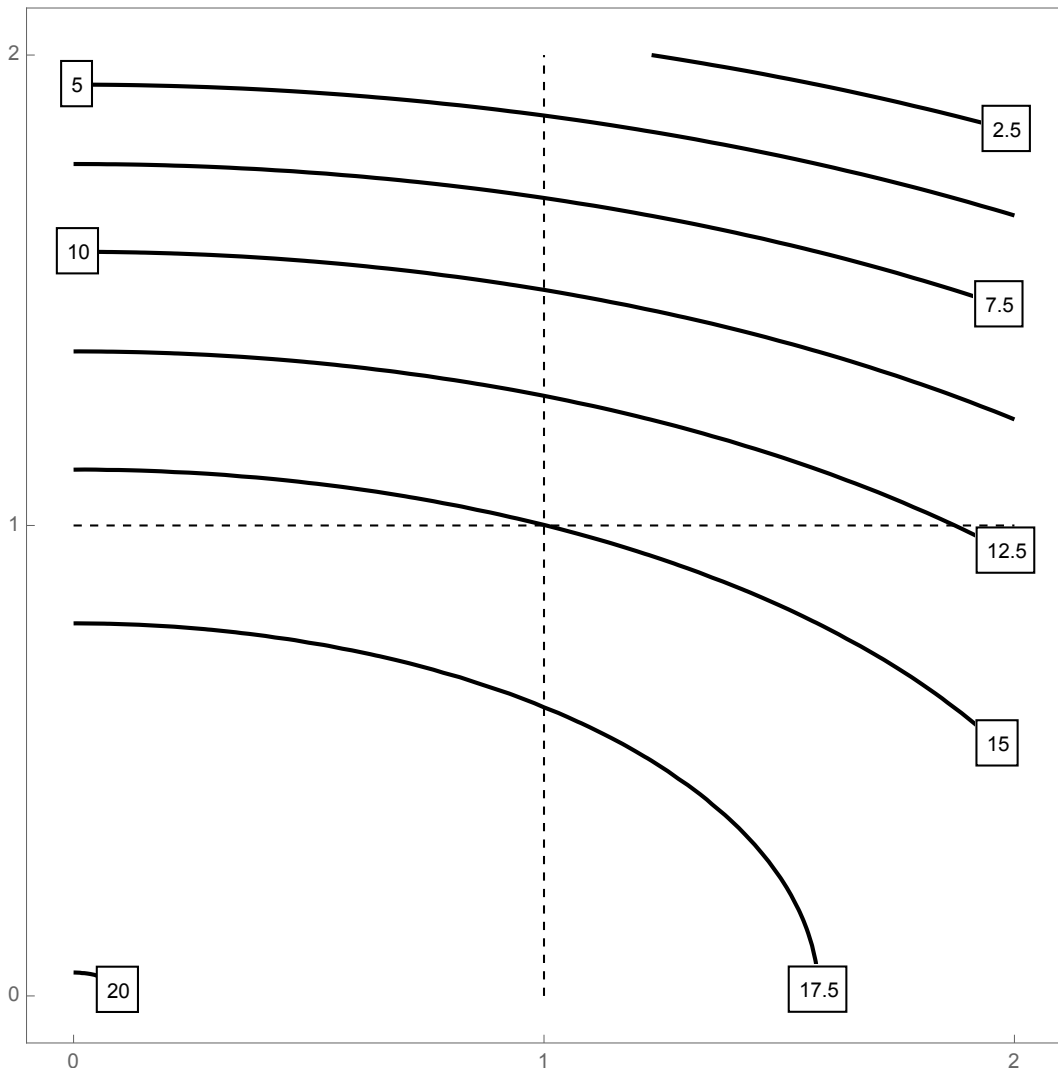


Figure 3: A contour plot with labels