Calc III - Final Review

Exam I

1. The equation
$$x = 2y + 3z + 4$$
describes a plane. Find
   (a) A point on the plane and
   (b) a normal vector for the plane.
2. An object moves according to the parametrization
$$\vec{p}(t) = \langle t^2, t^3, t^4 \rangle.$$
Find a parameterization of the line that is tangent to the path of motion of the object at the point where $t = 1$.
6. Let
$$\vec{p}(t) = \langle t+3, t+1, t+4 \rangle \text{ and } \vec{q}(t) = \langle t, 2t-3, 3t-1 \rangle.$$
Determine if these two lines intersect and, if so, find an equation of the plane containing the two lines.

Exam 2

3. Find the equation of the plane tangent to the graph of $x^2 + y^2 + z^3 = 1$ at the point $(1, 1, 1)$.
4. Find and classify the critical points of the function $f(x, y) = 6x^3 + 6xy + y^2$.
5. Use the method of Lagrange multipliers to find the extremes of $f(x, y) = x^2 + 4y^2$ subject to $x + 2y = 4$.
8. Figure 1 below shows the contour diagram of
$$f(x, y) = (x + y)e^{-2\left(x^2+y^2\right)}.$$
Identify the location or locations of any maxima that you see and find their exact location.

Exam 3

1. Evaluate the following multiple integrals.

   (a) $\int_0^1 \int_0^2 \int_0^1 x^2yz\,dx\,dy\,dz$

   (b) $\iint\limits_D 1\,dA$, where D is the region in the plane bound between $y = x^2 - 1$ and $y = x + 1$.

3. Find the volume trapped under the graph of the function $f(x, y) = 4 - \left(4x^2 + y^2\right)$ and over the xy-plane.

4. Let Q denote the quarter of a cylinder in the first octant with radius 1 and height 2 and centered on the z-axis as shown in figure 2. Set up the triple integral over Q of an arbitrary function $f(x, y, z)$

$$\iiint\limits_Q f(x, y, z)\,dx\,dy\,dz,$$

   as an iterated integral in cylindrical coordinates

5. Let T denote the top half of a sphere of radius 3 centered at the origin. Evaluate

$$\iiint\limits_T (x^2 + y^2 + z^2)^3\,dV.$$

A bit more

1. Let

$$\vec{F}(x, y) = \langle 2y, -x \rangle$$

   and let C be the curve parameterized by

$$\vec{r}(t) = \langle t^2, t^3 \rangle$$

   over the time interval $-1 \leq t \leq 1$. Compute

$$\int_C \vec{F} \cdot d\vec{r}.$$

2. Let $\vec{F}$ denote the conservative vector field

$$\vec{F}(x, y) = \left\langle 2xy^3 + 1, 3x^2y^2 + 1 \right\rangle.$$

   Find a potential function f for $\vec{F}$ and use it to compute

$$\int_C \vec{F} \cdot d\vec{r},$$

   where C is a path from the origin to the point $(1, 1)$.

3. Let
$$\vec{F}(x,y,z) = \langle x, y, z^2 \rangle.$$
Use the divergence theorem to compute
$$\int_S \vec{F} \cdot d\vec{n},$$
where S is the surface of the outward oriented unit cube.

4. Let E denote the positively ellipse $x^2 + 4y^2 = 4$.
   (a) Set up and evaluate the integral
   $$\oint_E 0\,dx + x\,dy.$$
   (b) Use Green's theorem to interpret your integral computation in terms of area.

5. The polygon shown in figure 3 has six vertices and six edges. Thus, I guess it's an irregular hexagon. It's edges are at the points
$$(\cos(i\pi/6), \sin(i\pi/6)), \quad \text{for} \quad i = 1 \ldots 6.$$
Write down a formula for the area of the polygon in terms of the vertices.
I think it might make a lot of sense to use a summation notation here.

6. Let $\vec{F}(x,y,z) = \langle 0, 0, z \rangle$ and let S denote
$$\{(\sin(\varphi)\cos(\theta), \sin(\varphi)\sin(\theta), \cos(\varphi)) : 0 \leq \varphi \leq \pi/6 \text{ and } 0 \leq \theta \leq 2\pi\}.$$
Thus, I guess that S looks something like our own Northern arctic region. Set up
$$\iint\limits_S \vec{F} \cdot d\vec{n}$$
as an iterated integral in terms of φ and θ.

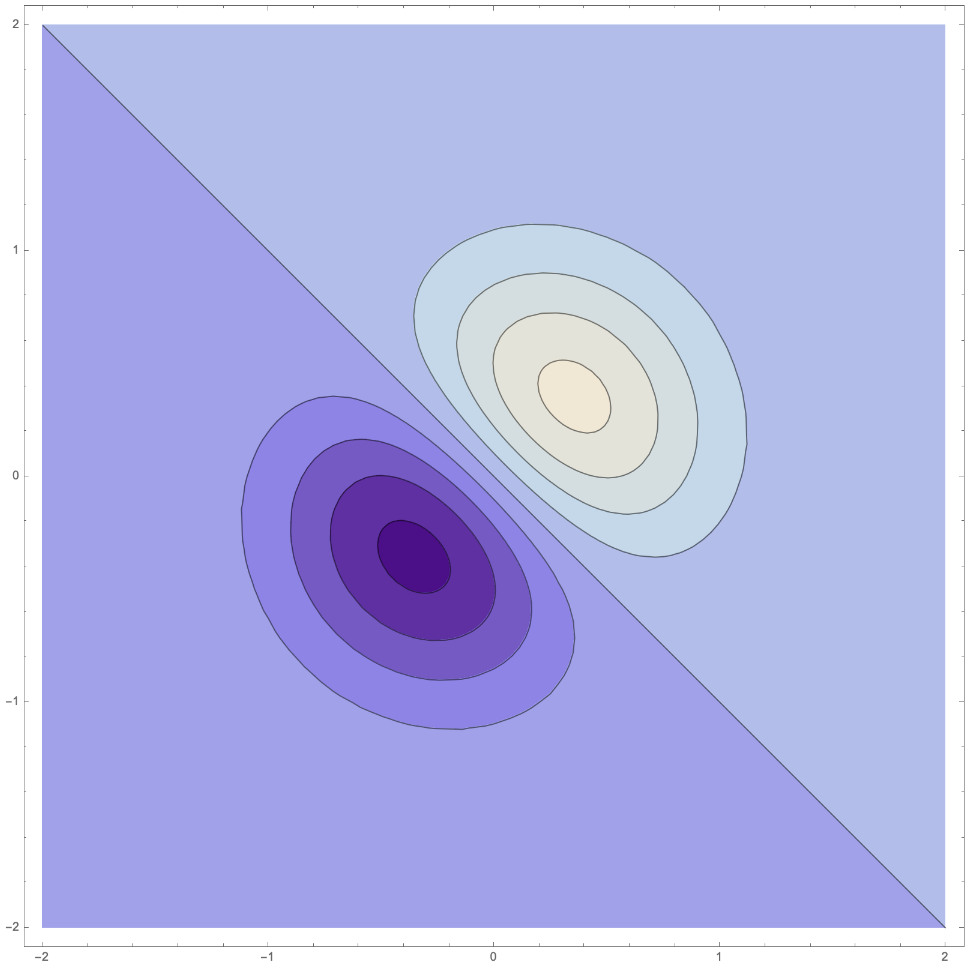

Figure 1: A contour diagram

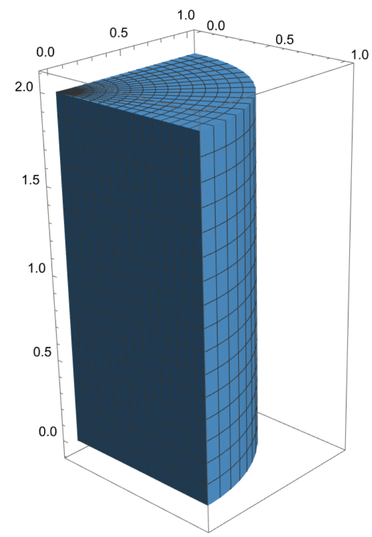

Figure 2: A quarter of a cylinder

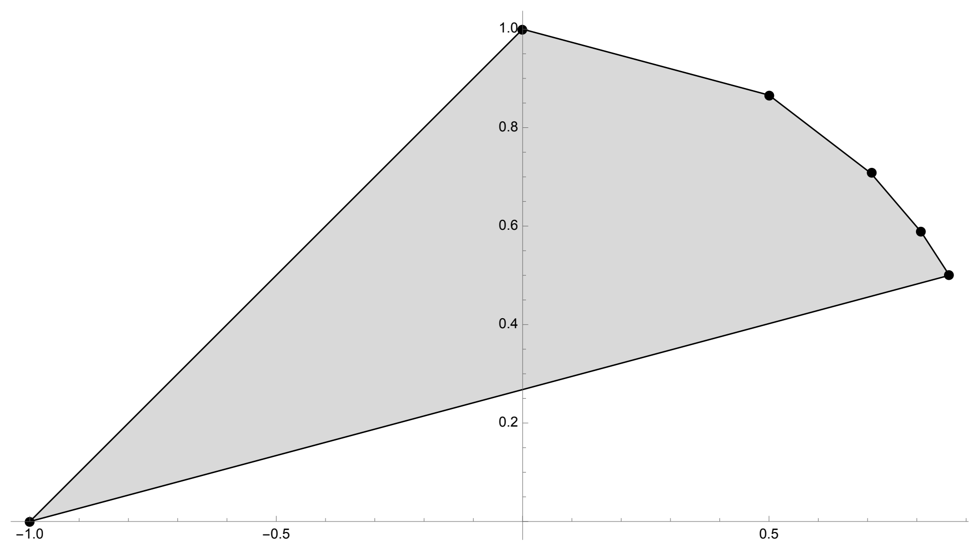

Figure 3: A hexagon