Calc II - Review for exam III

The third exam will be this Friday, April 15. We will discuss some of these problems in class on Wednesday, but you should work them all out to the best of your ability prior to that. Understanding the problems on this sheet will help you greatly on the exam.

1. Use the geometric series formula to express

$$\sum_{n=2}^{\infty}(-1)^n \frac{3^{n+1}}{5^{n-1}}$$

as a simple, finite combination of fractions.

2. Use the geometric series formula to express $0.21\overline{12}$ as a simple, finite combination of fractions.

3. Use the integral test to show that $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n(\ln(n))^p}$ converges precisely when $p > 1$.

4. Suppose we'd like to approximate

$$\sum_{n=1}^{\infty} \frac{n}{n^4+1}$$

by truncating the sum to obtain a finite sum of the form

$$\sum_{n=1}^{N} \frac{n}{n^4+1}.$$

How large does N have to be to ensure that our approximation is within 0.0001 of the actual value?

5. Suppose we'd like to approximate

$$\sum_{n=1}^{\infty}(-1)^{n+1} \frac{1}{n^4}$$

by truncating the sum to obtain a finite sum of the form

$$\sum_{n=1}^{N} \frac{n}{n^4+1}.$$

How large does N have to be to ensure that our approximation is within 0.0001 of the actual value?

6. Write down a couple of complete sentences using the comparison test to show that

$$\sum_{n=1}^{\infty} \frac{\sin\left(n^3\right)}{n^4}$$

converges absolutely.

7. Write down a couple complete sentences using the alternating series test to show that

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\ln(n)}{n}$$

converges conditionally.

8. Classify the following series as absolutely convergent, conditionally convergent, or divergent. Be sure to provide a clear and grammatically correct explanation.

   (a) $\sum(-1)^n\frac{n^2}{n^2+2}$

   (b) $\sum(-1)^n\frac{n^2}{n^3+2}$

   (c) $\sum(-1)^n\frac{n^2}{n^4+2}$

9. Suppose we take a square of side length one, break it into 9 copies scaled by the factor 1/3 and remove the one in the middle. We then do the same with the 8 remaining squares, and continue recursively, as illustrated in figure 1. The limit of the resulting figure is called the Sierpinski carpet. Find the total area of all the infinitely many squares that are removed in this process.

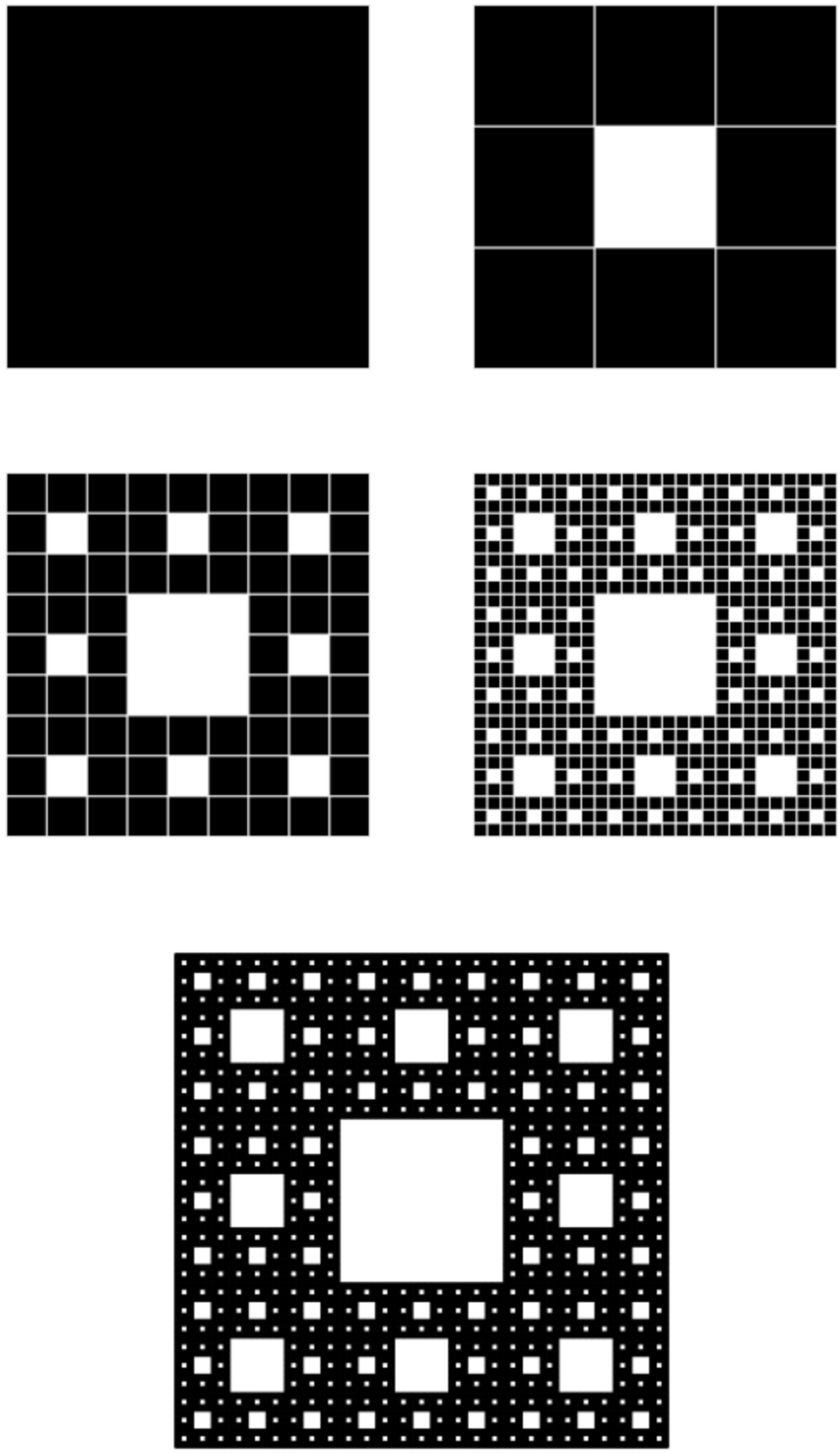

Figure 1: Approximations to the Sierpinski carpet