Calc II - Review for exam I

The first exam will be this Friday, February 4. We will discuss some of these problems in class on Wednesday, but you should work them all out to the best of your ability prior to that. Understanding the problems on this sheet will help you greatly on the exam.

1. Evaluate the following integrals using the technique indicated.
   (a) $\int \frac{x}{2x^2+1}\,dx$ - with u-subs
   (b) $\int x\cos(x)dx$ - by parts
   (c) $\int \cos^2(x)\,dx$ - with the $\frac{1}{2}$-angle formula
   (d) $\int \cos^3(x)\,dx$ - with u-subs
2. The complete graph of a function f is shown in figure 1 - it consists of two straight line segments and a quarter circle. Evaluate $\int_0^5 f(x)\,dx$.
3. Consider the integral $\int_0^2 x\sin\left(\cos\left(x^2+1\right)\right)dx$. Letting $u = x^2+1$, translate this integral to a new integral in terms of u. You need not evaluate the integral.
4. Suppose we know that
   $$\int_{-3}^{3} f(x)dx = 6.$$
   Which of the following integrals can we compute and what is the value?
   (a) $\int_{-1}^{1} f(3x)dx$
   (b) $\int_0^2 f(3x-3)dx$
   (c) $\int_0^3 f(x)dx$
5. Suppose we wish to estimate
   $$\int_0^3 \sqrt{x^3+1}dx$$
   with an approximating sum and we'd like our result to be within 0.0001 of the actual value.
   (a) Find an n large enough so that n terms will guarantee your estimate is within the desired accuracy.
   (b) Write down the resulting sum using summation notation.
   (c) Is your sum an upper bound or a lower bound?

   Note that you should think about this problem using each of our four types of approximating sums. Also note that the graph of $f(x) = \sqrt{x^3+1}$, together with its first and second derivatives, is show in figure 2.

6. Evaluate the following integrals using any technique that you see fit.

   (a) $\displaystyle\int_0^2 \sqrt{4-x^2}\,dx$

   (b) $\displaystyle\int x\sqrt{4-x^2}\,dx$

   (c) $\displaystyle\int_{-\pi}^{\pi} \frac{\sin^3(x)}{\cos^5(x)}\,dx$

   (d) $\displaystyle\int_0^{2\pi} \cos^2(x)\sin^2(x)\,dx$

   (e) $\displaystyle\int \frac{1}{x^2\sqrt{4-x^2}}\,dx$

7. Evaluate the following improper integrals, or state why they diverge.

   (a) $\displaystyle\int_0^{\infty} e^{-2x}\,dx$

   (b) $\displaystyle\int_1^{\infty} \frac{\ln(x)}{x}\,dx$

   (c) $\displaystyle\int_1^{\infty} \frac{1}{x^3}\,dx$

8. Write down a complete sentence proving that the improper integral $\displaystyle\int_1^{\infty} e^{-x^2}\,dx$ converges.

   Note: You may assume that $\displaystyle\int_1^{\infty} e^{-x}dx$ converges.

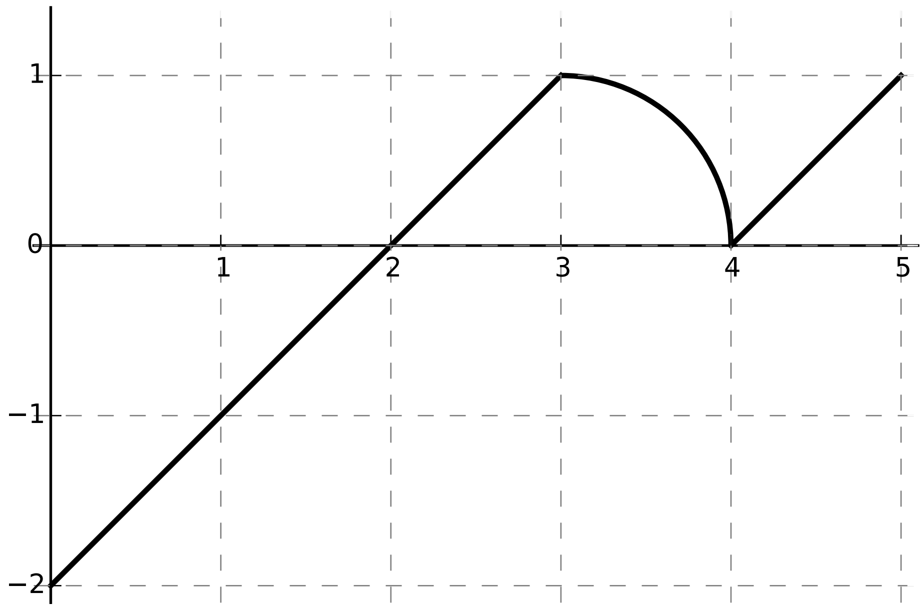

Figure 1: The complete graph of a function

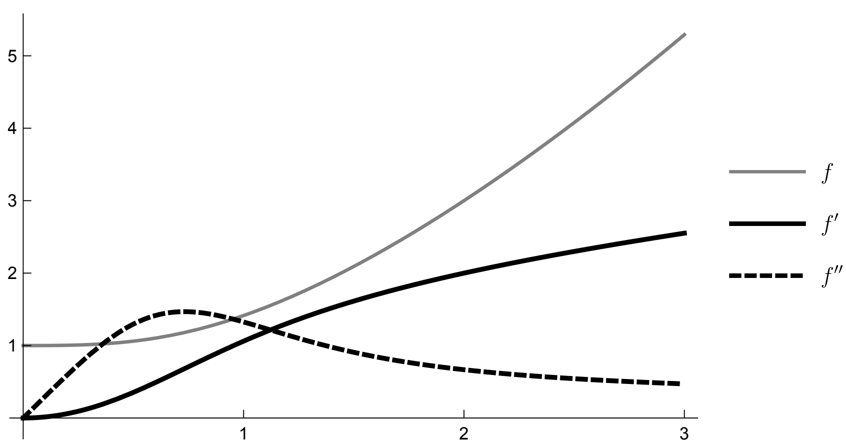

Figure 2: The graph of $f(x) = \sqrt{x^3 + 1}$ with its derivatives