Calc III - Review 2

1. Find the TNB frame for $\vec{p}(t) = \langle 2\sin(3t), 0, 2\cos(3t)\rangle$,
2. Find the TNB frame for $\vec{p}(t) = \left\langle \cos\left(t^2\right), t^2, \sin\left(t^2\right)\right\rangle$
3. Find the arc length of one "coil" of the helix:
4. Find the TNB frame for $\vec{p}(t) = \langle 2\sin(3t), t, 2\cos(3t)\rangle$.
5. Let $f(x, y) = xy^3$.
   (a) Compute ∇f.
   (b) From the point $(2, 1)$, in what direction $\boldsymbol{u}$ is $f$changing the fastest?
   (c) From the point $(2, 1)$, is there any direction $\boldsymbol{u}$ so that $D_{\boldsymbol{u}}f(2, 1) = 10$?
6. Find and classify the critical points of the function $f(x, y) = x^2 - 3xy + y^3$.
7. Find and classify the critical points of the function $f(x, y) = x^3 - 2x^2 + xy + y^2 - 3x$.
8. In this problem, we'll use the method of Lagrange mulitipliers to find the extremes of $f(x, y) = x + y^2$ subject to the constraint $x^2 + y^2 = 1$.
   (a) Sketch the contour diagram of f together with the constraint curve. The contour diagram of f should be a collection of parabolas and the constraint curve should be a circle. As a contour of f corresponds to an equaiton of the form $f(x, y) = c$, be sure to label each contour with its c value.
   (b) Use your contour diagram to find the approximate location of the absolute minimum and absolute maximum of f along the constraint curve.
   (c) Use the method of Lagrange mulitipliers to find the exact locations of the points you found in part (b).
9. Find the equation of the plane tangent to the graph of $2x^2 - y^2 + z^2 = 8$ at the point $(2, 1, 1)$.
10. Let $f(x, y) = 3(x - 2)^2 + (y + 1)^2$.
    (a) Sketch and label several contours of f.
    (b) What does part (a) tell you about local maximum and minimum values of f?
    (c) Sketch the gradient field of f over your contour diagram. Pay special attention to the direction an relative magnitudes of the gradient vectors.
11. Use the method of Lagrange multipliers to find the extremes of $f(x, y) = 2x + 4y$ subject to $x^2 + y^2 = 20$.

12. Let $f(x,y,z) = y^2 z - x^2 y$.

    (a) Compute ∇f.

    (b) Find an equation of the plane tangent to the surface $f(x,y,z) = 10$ at the point $(1,2,3)$.

13. Match the following functions with the graph or contour plot in figure 1.

    (a) $f(x,y) = 2x^2 + y^2 + x^2 y + 4$

    (b) $f(x,y) = e^{-\left(x^2+y^2\right)}$

    (c) $f(x,y) = \cos\left(x^2+y^2\right)$

    (d) $f(x,y) = x^2 - y^2$

14. Note that the contour plot labeled IV in the figure 1 has several points lying on contour lines. Sketch the gradient vectors at those points. Be sure to pay attention to the direction and relative magnitudes of those vectors.

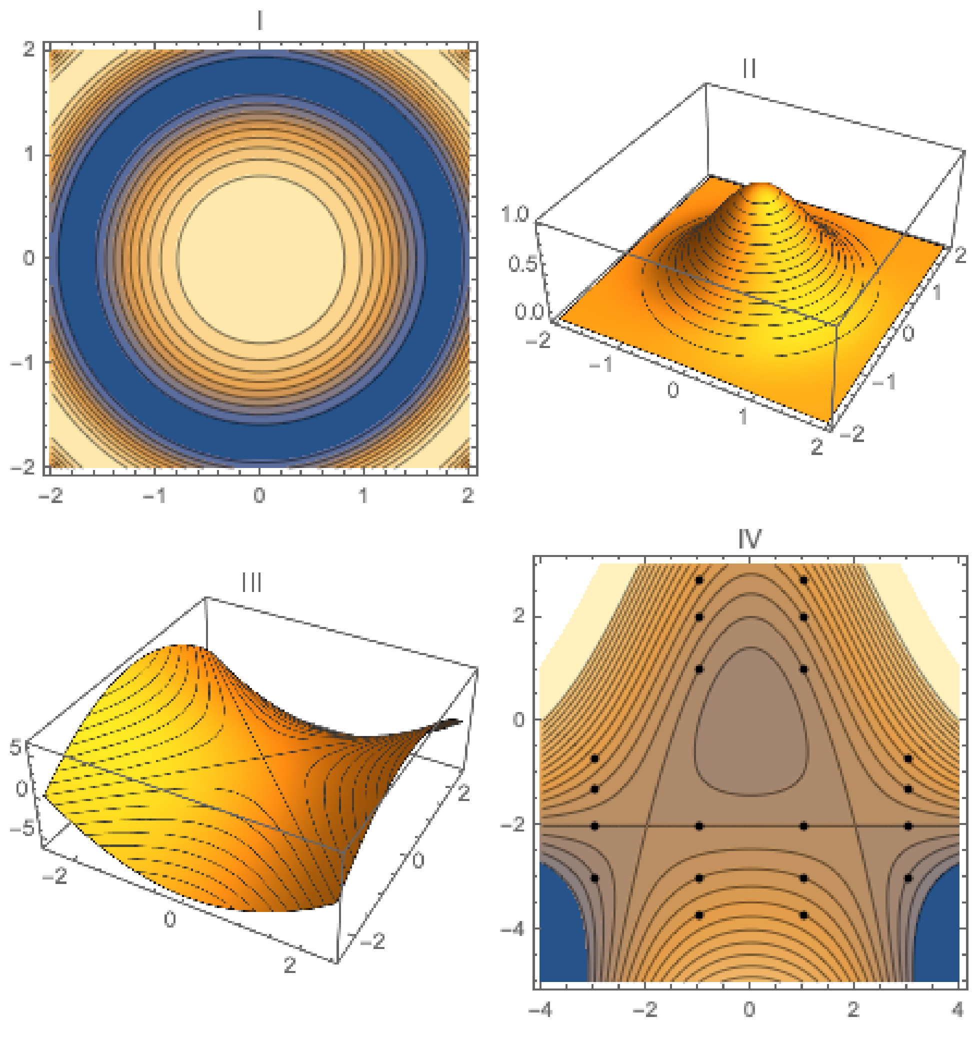

Figure 1: Some groovy pictures