Calc I - Review for Exam III

There will be an exam This Friday, April 13 and many of the problems will be like something on this review sheet.

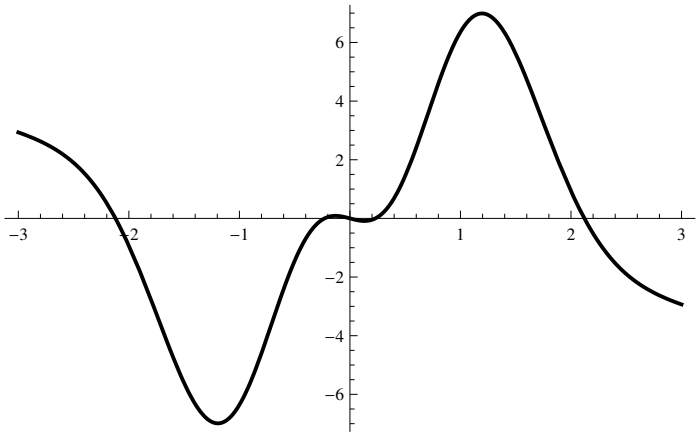

Figure 1: The graph of $f(x) = 20x^3e^{-x^2} - x$

1. A few more derivatives
   (a) $f(x) = e^x + \ln(x)$
   (b) $f(x) = 4x + 2\tan(x^2)$
   (c) $f(x) = 2x + \log(x)\arctan(\ln(x))$
   (d) $f(x) = \frac{4x}{3} - e^x + \arcsin(x^3 - 1)$
   (e) $f(x) = \arcsin^2(2x + 1)e^{-3x}$
2. Find an equation of the line that is tangent to the curve satisfying $x^3 - 3x^2y^3 + y^4 = -1$ at the point $(1, 1)$.
3. In this problem, we're going to derive the fact that, if $f(x) = \ln(2x)$, then $f'(x) = 1/x$ using the fact that we know the inverse of f.
   (a) Starting from $y = \ln(2x)$, write the equation in exponential form.
   (b) Implicitly differentiate your equation from part (a) with respect to x.
   (c) Solve your equation from part (b) for y'.
   (d) Simplify, if necessary to show that $y' = 1/x$.

4. The graph of $f(x) = 20x^3e^{-x^2} - x$ is shown in figure 1.
   (a) Write down an equation that the critical points of f must satisfy.
   (b) Suppose we wanted to find the smallest, positive critical point of f using Newton's method. Write down the corresponding Newton's method iteration function and a reasonable initial guess to start the iteration.
   (c) Find the exact values of the inflection points of f and indicate their positions in the graph.
5. Let $f(x) = 3x^4 - 4x^3 - 12x^2$
   (a) Find all the critical points of f.
   (b) Sketch a rough graph of f.
   (c) Find the absolute minimum value of f.
   (d) Find the locations of any other local extremes of f.
6. The top of a 15 foot long ladder slides down a wall at 3 feet per second. How fast is the bottom of the ladder moving away from the wall when it is 12 feet away from the wall?
7. Suppose I set up a rectangular corral to enclose 1000 square feet with inner partitions, as shown in figure 2. The material for the exterior portion costs twice as much as the material for the interior walls. What are the dimensions of the cheapest such corral?
   Comment: There's a good chance that a familiar optimization problem - like this one or a pizza box type problem will be on the exam.
8. Let $f(x) = x^3 - x - 1$. Use two Newton steps from $x_0 = 1$ to find a good rational approximation to the root of f.

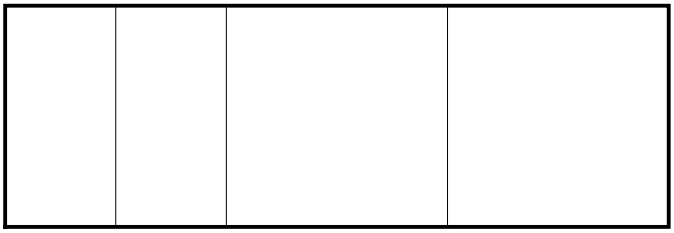

Figure 2: A corral