Calc I - Review for exam I

The first exam will be next Friday, February 16. We will discuss some of these problems in class on Wednesday, but you should work them all out to the best of your ability prior to that. Understanding the problems on this sheet will help you greatly on the exam.

Table 1: Usain Bolt's splits for the the 2008 Olympic 100m race

0-10m	10-20m	20-30m	30-40m	40-50m	50-60m	60-0m	70-80m	80-90m	90-100m	Total
1.85	1.02	0.91	0.87	0.85	0.82	0.82	0.82	0.83	0.90	9.69

1. Table 1 shows the splits for Usain Bolt's 100m race at the 2008 Olympics.
   (a) What was his average speed for the race in meters per second?
   (b) What is your best estimate his top speed?
2. Let $f(x) = 3 + 2x - x^2$
   (a) Sketch a rough graph of f.
   Be sure to clearly indicate the locations of the vertex and y-intercepts.
   (b) Find the x-intercepts. Label them on your graph.
   (c) Draw the secant line that goes through the vertex and the smaller of the two x-intercepts.
   (d) Find an equation for your secant line.
3. Let $f(x) = x^2 - 4x$. Our mission is to find an equation of the line that is tangent to the graph of f at the point where $x = 3$.
   (a) Sketch a rough graph of f.
   Be sure to clearly indicate the locations of the vertex and y-intercepts.
   (b) Draw the tangent line at $x = 3$. Based on your drawing, do you expect a positive slope or a negative slope?
   (c) Write down the difference quotient for f at the point $x = 3$. Simplify your difference quotient to the point where you can cancel the h in the denominator.
   (d) Evaluate the limit as $h \to 0$ of your difference quotient. What does this result tell you about your tangent line?
   (e) Write down an equation for the tangent line.
4. Use the limit laws to show that $\lim_{x\to 2} x\left(x^2+2\right)$, *showing every detail.*
5. Let
$$f(x) = \frac{(10x-2)(x+4)}{x^2-16}.$$
   Evaluate each of the following the limits.
   (a) $\lim_{x\to -4} f(x)$
   (b) $\lim_{x\to 4^-} f(x)$
   (c) $\lim_{x\to 4^+} f(x)$
   (d) $\lim_{x\to 4} f(x)$
   (e) $\lim_{x\to 1} f(x)$

6. The table below shows the function $f(h) = \left(5^h - 1\right) / h$ evaluated at several values of h near zero. Use this table to obtain a numerical estimate of $\lim_{h \to 0} f(h)$. How many digits of your estimate do you believe are accurate?

h	0.1	0.01	0.001	0.0001	0.00001
$f(h)$	1.74619	1.62246	1.61073	1.60957	1.60945

7. Evaluate each of the following the limits.

   (a) $\displaystyle\lim_{x \to -2} \frac{x^2 - 4}{x + 2}$

   (b) $\displaystyle\lim_{x \to 2} \frac{x^2 - 4}{x + 2}$

   (c) $\displaystyle\lim_{x \to -2} \frac{x - 4}{x + 2}$

   (d) $\displaystyle\lim_{x \to 3} \frac{x - 3}{x^3 - 2x^2 - 2x - 3}$

   (e) $\displaystyle\lim_{x \to 4} \frac{1}{2 - \sqrt{x}} \left(\frac{1}{\sqrt{x}} - \frac{1}{2} \right)$

8. Write down the definition of continuity.

9. Suppose that $f(x) = x^5 + 2x - 1$. Prove that f has a root in the interval $[0, 1]$.

10. Figure 1 shows the complete graph of a function f.

    (a) What is the domain of f?

    (b) What is the set of points of continuity of f?

    (c) What are $f(1.5)$ and $\lim_{x \to 1.5} f(x)$?

    (d) What are $\lim_{x \to 1^-} f(x)$ and $\lim_{x \to 1^+} f(x)$? What does this say about $\lim_{x \to 1} f(x)$?

    (e) How do you know that f has a root between $x = 1.6$ and $x = 1.9$? (Your answer should involve the intermediate value theorem.)

    (f) Note that $f(0.9) > 0$ and $f(1.1) < 0$ but f has no root between 0.9 and 1.1. Why does this not violate the intermediate value theorem?

11. Find the derivatives of the following functions, *using the definition of the derivative.*

    (a) $f(x) = 3x^2 - x$

    (b) $f(x) = 2x^3$

12. Figure 2 shows the complete graph of a function together with a spare set of axes. Sketch a graph of f' on that spare set.

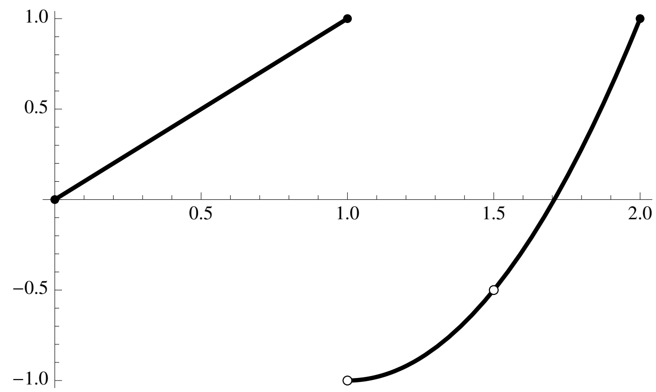

Figure 1: The complete graph of a function

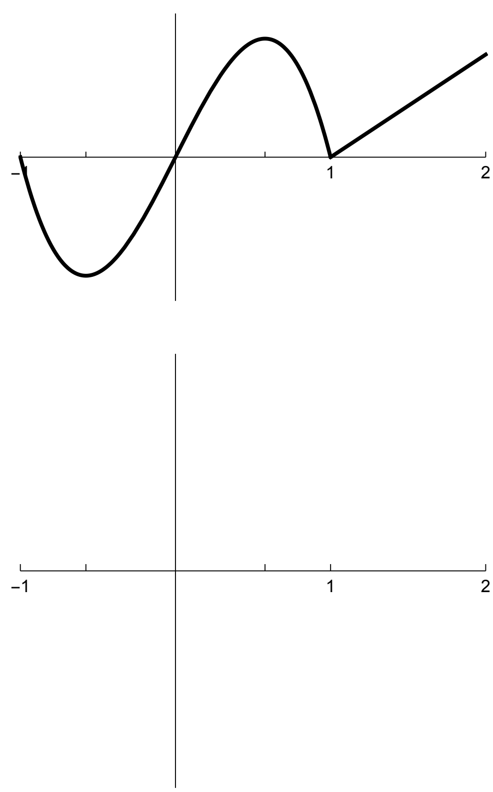

Figure 2: The complete graph of a function together with a spare set of axes