Calc I - Review for Exam II

Our next exam will be this Friday, March 4 and many of the problems will be like something on this review sheet.

1. Use the definition of the derivative to prove this special case of the product rule:
$$\frac{d}{dx}x^2 f(x) = 2xf(x) + x^2 f'(x).$$

2. Use the differentiation rules to find f' for each of the following functions.
   You should do *lots* of these.
   (a) $y = x^2 - 2x - 3$
   (b) $y = \sin(x) + \cos(x) + \ln(x) + e^x$
   (c) $y = \sin(x)\cos(x)\ln(x)e^x$
   (d) $y = \sin\left(\ln\left(\cos(x)e^x\right)\right)$
   (e) $y = x^{2x}$
   (f) $y = \left(x + x^{-1}\right)\sqrt{x+1}$
   (g) $y = \left(\cos(6x) + \sin\left(x^2\right)\right)^{1/3}$
   (h) $y = e^{\ln(\cos(x)\tan(x))}$

3. Prove the following differentiation rules using a simpler differentiation rule.
   (a) $\frac{d}{dx}\tan(x) = \sec^2(x)$
   (b) $\frac{d}{dx}a^x = a^x \ln(a)$

4. Write the equation $y = \arccos(x)$ in terms of the cosine and use implicit differentiation to compute y'.

5. Find an equation of the line tangent to the curve $x^3y^2 + 2x^3y^4 = 3$ at the point $(1, 1)$.

6. Find the location and value of all local maxima and minima of $f(x) = 2x^3 + 3x^2 - 12x + 1$.

7. Find the absolute maximum and minimum of $f(x) = x^3 e^{-\frac{1}{2}x^4}$ over the interval $[-1, 2]$.

8. In this problem, we'll explore the possibility that $\frac{d}{dx}\sin(x) = \cos(x)$.
   (a) Write down the difference quotient for $f(x) = \sin(x)$ and set $x = 0$.
   (b) Compute the limit as $h \to 0$ of your answer from part (a).
   Note: You may assume the facts that $\lim_{\theta\to 0}\sin(\theta)/\theta = 1$ and that $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$.
   (c) Using the graph of the sine and the spare set of axes shown in figure 1, sketch a rough graph of the derivative of the sine function. Be sure to take the exact value computed in part (b) into account.

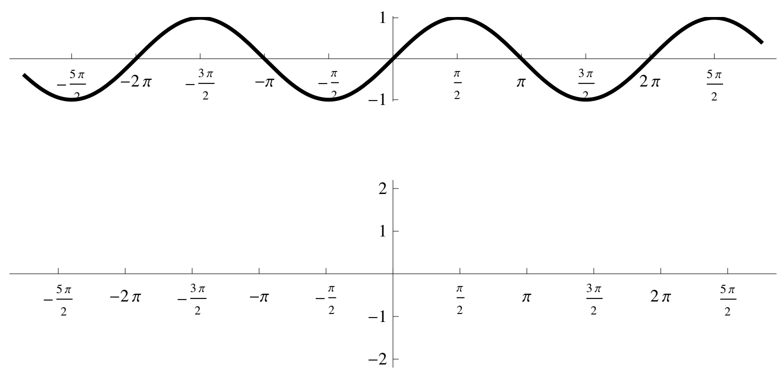

Figure 1: The sine function and a spare set of axes