Review for Exam 2

2026-11-11

Our next exam is next Wednesday, November 18. Here are some topics and problems to study.

Definitions and theorems

1. Riemann zeta function
2. Riemann hypothesis
3. The Cauchy-Gorsat Theorem
4. The Cauch Integral Formula
5. Attractive fixed point of a function $f : \mathbb{C} \to \mathbb{C}$.
   (Can you rephrase it for $\mathbb{C}$, rather than $\mathbb{R}$?)

Problems

1. Express the function $S(z)$ given by

$$S(z) = \sum_{n=3}^{\infty} \frac{(z-2)^n}{2^{n+1}}$$

   as a closed form function of z. What is the largest open disk on which your function is equal to $S(z)$?

2. Express

$$\int \frac{\sin(z)}{z}\, dz$$

   as a power series.

3. Show that

$$\sum_{n=1}^{\infty} \frac{1}{n^z}$$

   converges for $\mathrm{Re}(z) > 1$.

4. Suppose I tell you that

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{1/3}} \approx 0.571753.$$

   Can you give me a good numerical approximation to $\zeta(1/3)$?

5. Let C denote curve that follows the right half of the unit circle from $-i$ to i. Set up and evaluate

$$\int_C \frac{1}{z^5} dz.$$

6. Let C denote the positively oriented circle of radius 2 centered at the origin. Evaluate

$$\int_C f(z)\, dz$$

   for each of the following functions:
   a. $f(z) = 1/(z-1)$
   b. $f(z) = 1/(z-3)$
   c. $f(z) = z/((z-1)(z-3))$
   d. $f(z) = \sin(z)/z$
   e. $f(z) = \cos(z)/z$

7. Suppose that $f(z) = z^2 - 2i = (z-(1+i))(z+(1+i))$.
   a. Write down the Newton's method iteration function N for f.
   b. Suppose we iterate N starting from $z_0 = 10$. What is the limit of the sequence that we generate?

8. Find and classify the fixed points of $f(z) = z(2-z)$