

More practice for Exam 1

Here's a bit more practice for our first exam this Wednesday. This does *not* replace the review sheet and you should definitely study that as well. The exam will certainly have a few definition statements on it, for example.

1. Let $\sqrt[3]{x}$ denote the *principal* cube root of x . Evaluate

- a. $\sqrt[3]{8}$ and
- b. $-\sqrt[3]{-8}$

Express these in both Cartesian and polar form.

2. Use the Cauchy-Riemann equations to determine which of the following defines an analytic function.

- a. $f(x, y) = (x + x^2 - y^2) + i(y + 2xy)$
- b. $g(x, y) = (x + x^2 - y^2) + i(y - 2xy)$

3. Write down an ε - δ proof of the fact that

$$\lim_{z \rightarrow -i} ((3 + 4i)z + 2) = 6 - 3i.$$

4. Let C be the circle containing 2, -2 , and $2i$ and let

$$T(z) = \frac{z + 2}{z - 2}.$$

Describe the image of C under the action of T . What is the image of the interior of the circle?

5. Suppose I'd like a Möbius transformation that maps the points

$$(i, 1, 2) \text{ to } (-1, -i, 4)$$

in that order. Write down a specific equation in terms of variables w and z that you could solve to find that Möbius transformation and a brief description of why it should work. You do not need to solve the equation.

6. Let

$$f(z) = \frac{3iz^3}{z^2 - i}.$$

We consider f as a map of the Riemann sphere to itself that fixes ∞ . Compute $f'(\infty)$ and interpret the result.

7. Describe the image of the vertical line $\{z \in \mathbb{C} : \operatorname{Re}(z) = 1\}$ under the action of the exponential function.