

Review for quiz 1

2025-08-30

We have our first quiz this Friday, September 4th. This problem sheet represents most of the types problems that will be on the quiz. You should absolutely study it fully and completely!

Problems

1. Curious about the following limit,

$$\lim_{x \rightarrow 0} (1 + x)^{3/x},$$

I used my computer to plug in several values of x that are *close* to 0 but not equal to 0. The results are shown in Table 1 below.

Table 1: Values of $f(x) = (1 + x)^{3/x}$ near $x = 0$.

x	0.100000	0.010000	0.001000	0.000100	0.000010
$f(x)$	17.449402	19.788466	20.055451	20.082525	20.085236

Based on those computations, can you make a conjecture as to the approximate value of the limit? Be sure to indicate how many digits you believe to be correct and why.

2. The graph of

$$f(x) = \frac{x - 1}{x^3 - x^2 + x - 1}$$

is shown in Figure 1.

- Judging from the figure, what do you suppose is the value of $\lim_{x \rightarrow 1} f(x)$?
 - Use a little algebra together with the limit laws to prove that your guess is correct. Be sure to write you answer out using one or two complete sentences using proper mathematical syntax and connectives.
3. The Complete graph of a function f is shown in Figure 2. At each of the points $a = -1$, $a = 1$, $a = 2$ and $a = 4$, find the value of

- a) $f(a)$,
- b) $\lim_{x \rightarrow a^-} f(x)$,
- c) $\lim_{x \rightarrow a^+} f(x)$, and
- d) $\lim_{x \rightarrow a} f(x)$.

4. Compute the following limits quickly.

- a) $\lim_{x \rightarrow 4} \frac{2x^2 - 3x - 2}{x - 2}$
- b) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$
- c) $\lim_{x \rightarrow 3} \frac{x^2 - 6}{x - 3}$
- d) $\lim_{x \rightarrow 3} \frac{x^2 - 6}{(x - 3)^2}$
- e) $\lim_{x \rightarrow \infty} \frac{x^2 - 6}{x - 3}$
- f) $\lim_{x \rightarrow \infty} \frac{x^2 - 6}{(x - 3)^2}$

5. Let $f(x) = x^3 - x - 2$. Write a complete sentence referring to the intermediate value theorem *proving* that f has a root between $x = 0$ and $x = 2$.

Figures

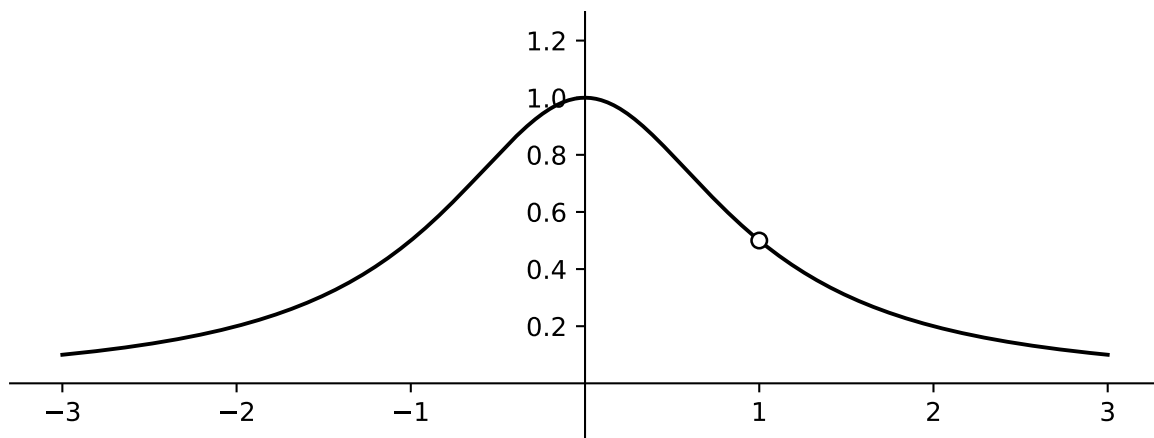


Figure 1: The graph of $f(x) = (x - 1)/(x^3 - x^2 + x - 1)$

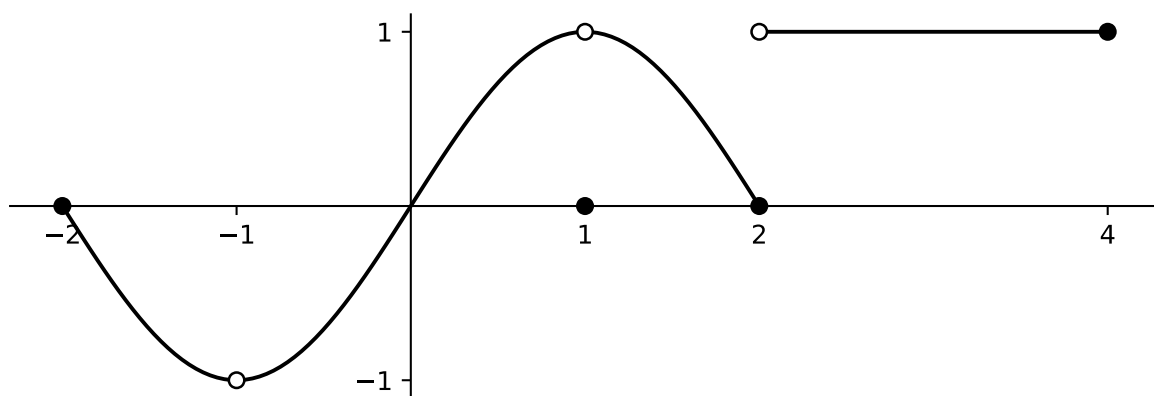


Figure 2: A graph for limits