Review for exam 3

We have our last midterm this coming Friday, November 20th. The problems on the exam will be very much like the problems you see here. I will go over this problem sheet in class on Wednesday but it will help you immensely to think about it on your own first so *please* work it out to the best of your ability prior to meeting on Wednesday.

Problems

1. The graph of $f(x) = x\ln(x)$ is shown in figure Figure 1. Find the exact location of the global minimum.
2. The graph of $f(x) = (x^2 - e^{-2x})/2$ is shown in Figure 2. Find the exact coordinates of any inflection point that you see.
3. I need to build a box with a square base and no top that needs to hold 500 cm^3. What's the minimal surface area required to accomplish this?
4. Let $f(x) = x^3 - 8$.
   - Write down the Newton's method iteration function $N(x)$ for f.
   - Apply $N(x)$ to take two Newton steps from $x_0 = 1$.
5. Figure 3 shows the data
   $$\{(-3,-2),(-2,1),(2,-1),(3,2)\},$$
   together with its regression line of the form $f(x) = ax$.
   a. Write down the formula for $E(a)$ representing the squared error of the regression as a function of the parameter a.
   b. Find the value of a that minimizes that squared error.
6. Write down both right and left Riemann sums for
   $$\int_0^2 e^x\,dx$$
   using $n = 4$ terms. Which is an upper bound and which is a lower bound for the actual value?
7. The complete graph of a function f is shown in Figure 4. Evaluate
   $$\int_{-4}^{4} f(x)\,dx.$$
8. An object is thrown off a 90 ft tall cliff with an initial vertical velocity of 20 ft/s. Find the function $y(t)$ that describes the height of the object as a function of time t.

9. Evaluate the following definite and indefinite integrals.
   a. $\int (x^2 + e^x - \sin(x) + \cos(x))\, dx$
   b. $\int \frac{x^3 + x^2 + x + 1}{x^2}\, dx$
   c. $\int x \sin(x^2 + 1)\, dx$
   d. $\int_0^2 (x^2 + 1)\, dx$
   e. $\int_0^2 x\sqrt{x^2 + 1}\, dx$

Figures

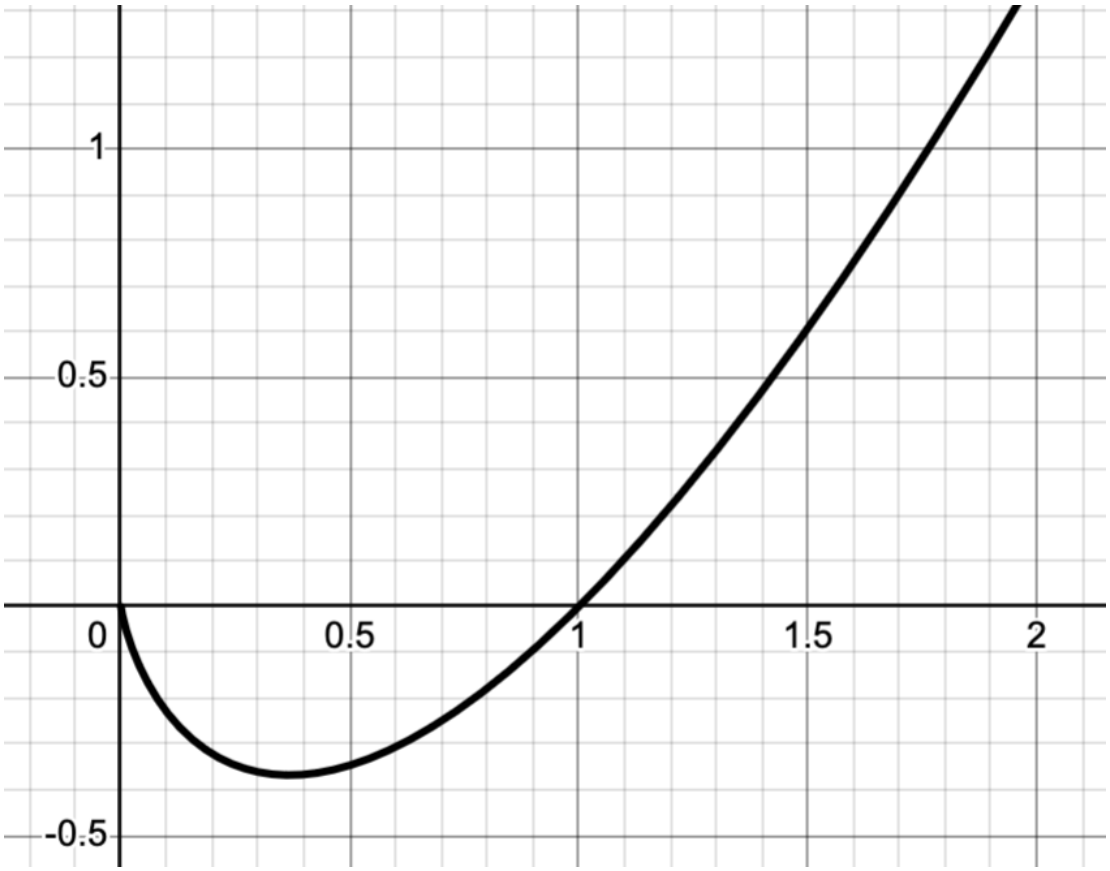


Figure 1: The graph of $f(x) = x \ln(x)$

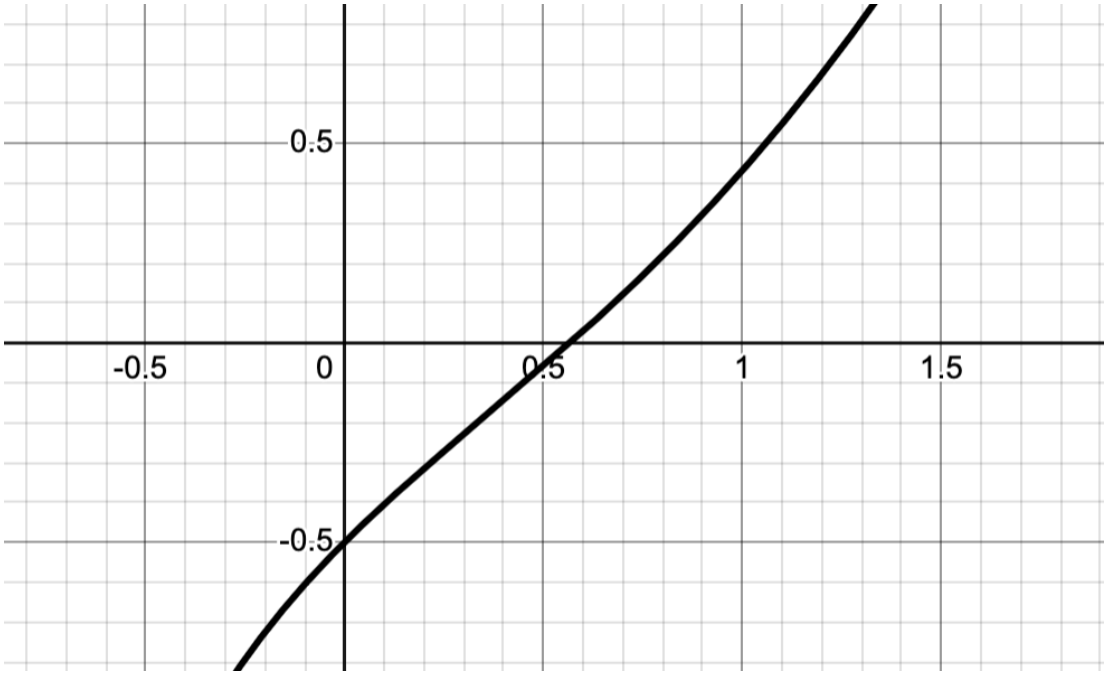


Figure 2: The graph of $f(x) = (x^2 - e^{-2x})/2$

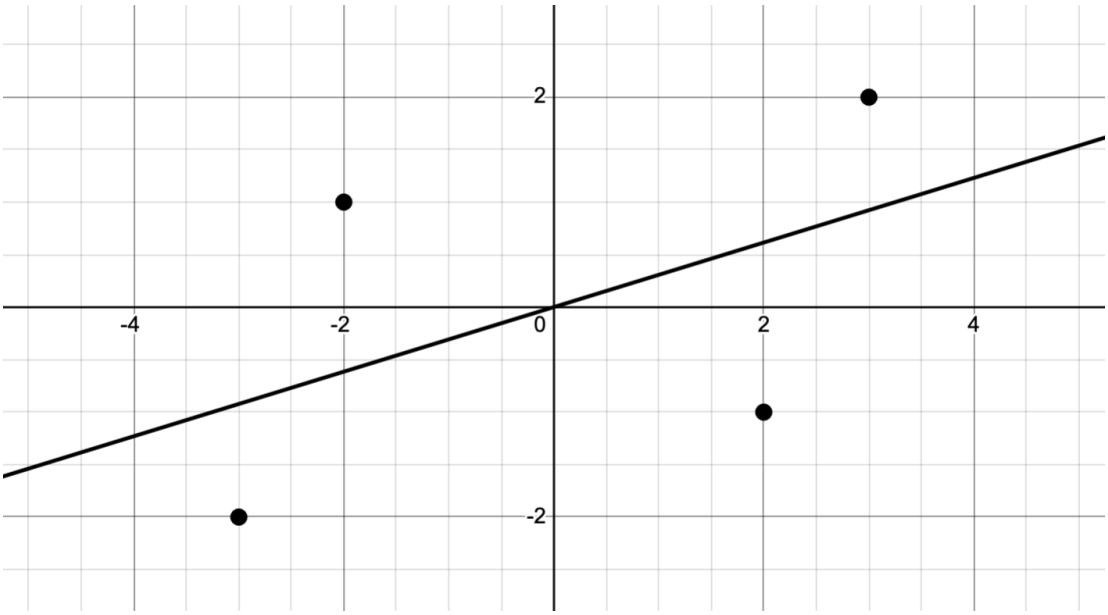


Figure 3: A little data together with its regression line

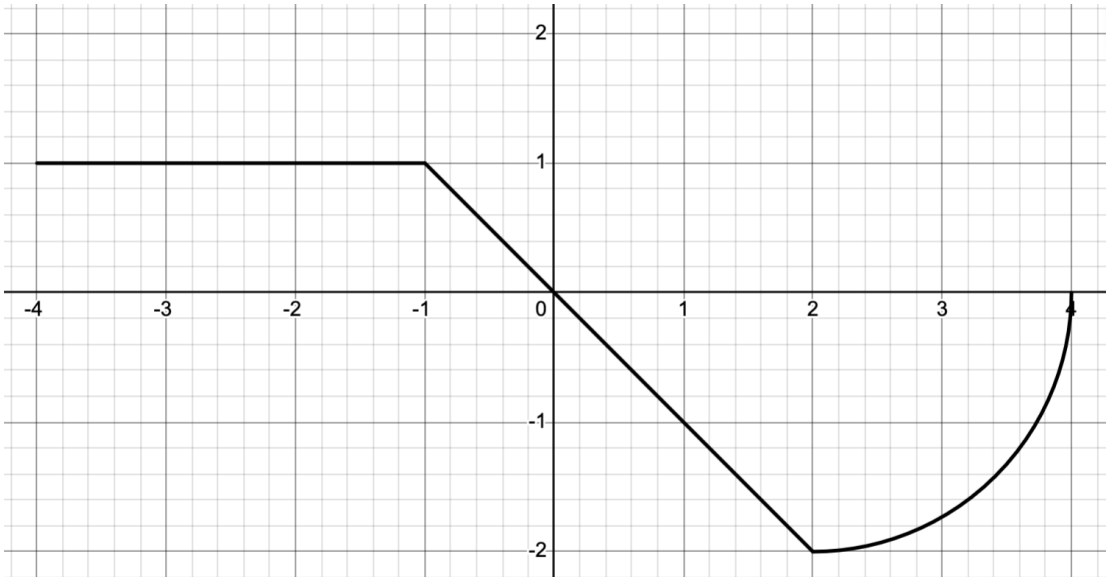


Figure 4: The complete graph of a function