Review for exam 1

We have our first exam next Friday, September 18th. The problems on the exam will be very much like the problems you see here. We will practice similar problems in class next Wednesday and it will help you immensely if you think this stuff out prior to that. You've got several resources to get help on this problem sheet, including:

- Our fabulous SI Leader Kris,
- The math lab, and
- Our forum, where I will offer direct help via the link at the bottom of the page.

Problems

1. Curious about the following limit,

$$\lim_{x\to 0} \frac{\tan\left(\sqrt{x/\pi}\right)}{\sqrt{x}},$$

I used my computer to plug in several values of x that are *close* to 0 but not equal to 0. The results are shown in Table 1 below.

Table 1: Values of $f(x) = \frac{\tan(\sqrt{x/\pi})}{\sqrt{x}}$ near $x = 0$.

x	0.1000000	0.0100000	0.0010000	0.0001000	0.0000100
$f(x)$	0.5702530	0.5647890	0.5642495	0.5641956	0.5641902

Based on those computations, can you make a conjecture as to the approximate value of the limit? Be sure to indicate how many digits you believe to be correct and why.

2. The graph of a function is shown in figure Figure 1.

   a) For $a = -3, -2, -1, 0,$ and 1, find
      i) $\lim_{x\to a} f(x)$
      ii) $\lim_{x\to a^+} f(x)$
      iii) $\lim_{x\to a^-} f(x)$, and
      iv) $f(a)$

   b) What are
      i) $\lim_{x\to -4^-} f(x)$ and
      ii) $\lim_{x\to \infty} f(x)$?

3. Let's suppose that we know that

$$\lim_{x\to 2} f(x) = 3 \text{ and } \lim_{x\to 2} g(x) = 5.$$

What can you say about the following limits?

a) $\lim_{x\to 2}(2f(x) - g(x))$?
b) $\lim_{x\to 2} \dfrac{f(x)-3}{g(x)-5}$?

4. Compute the following limits.

a) $\lim_{x\to 4} \dfrac{x^2-3x-4}{x-4}$
b) $\lim_{x\to -2} \dfrac{x^2+5x+6}{x+2}$
c) $\lim_{x\to 1} \dfrac{2x^2+x+5}{x+1}$

Note: There's likely to be one slightly harder problem with a bit more challenging algebra. Here are a couple of examples:

d. $\lim_{x\to -2} \dfrac{x+2}{x^3+3x^2+3x+2}$
e. $\lim_{x\to 3/2} \dfrac{2x-3}{2x^3-5x^2+x+3}$

5. Compute the following limits involving infinity.

a. $\lim_{x\to\infty} \dfrac{2x^2+3x-2}{5x^2-4}$
b. $\lim_{x\to\infty} \dfrac{2x^2+3x-2}{5x^3-4}$
c. $\lim_{x\to 2^-} \dfrac{x+1}{x-2}$

6. Write down a complete sentence referring to the intermediate value theorem explaining why $f(x) = 3x^7 - x - 1$ has a root between $x = 0$ and $x = 1$.

7. Find the derivatives of the following functions, *using the definition of the derivative*.

a. $f(x) = 3x^2 + 2x + 1$
b. $f(x) = x^6$
c. $f(x) = 1/\sqrt{x}$.

8. Suppose that f and g are differentiable functions. Show that

$$\frac{d}{dx}(3f(x) - g(x)) = 3f'(x) - g'(x),$$

using the definition of the derivative.

9. Find the derivatives of the following functions using the basic differentiation rules.

   a. $f(x) = 3x^2 + 2x + 1$
   b. $f(x) = 61x^{48} + 2x^{16} + x^{14} - x - 1$
   c. $f(x) = \frac{x^4 - x^3 + 2x^2 - x + 1}{x^2}$.
   d. $f(x) = \frac{x-1}{\sqrt{x}}$.

10. The graph of the function f is shown in the top part of Figure 2. Sketch the graph of f' on the axes provided below the graph of f.

11. Let $f(x) = 4 + 4x - x^2$.

    a. Sketch the graph of f, together with the its tangent line at the point $x = 1$.
    b. Find a formula for the tangent line.

Figures

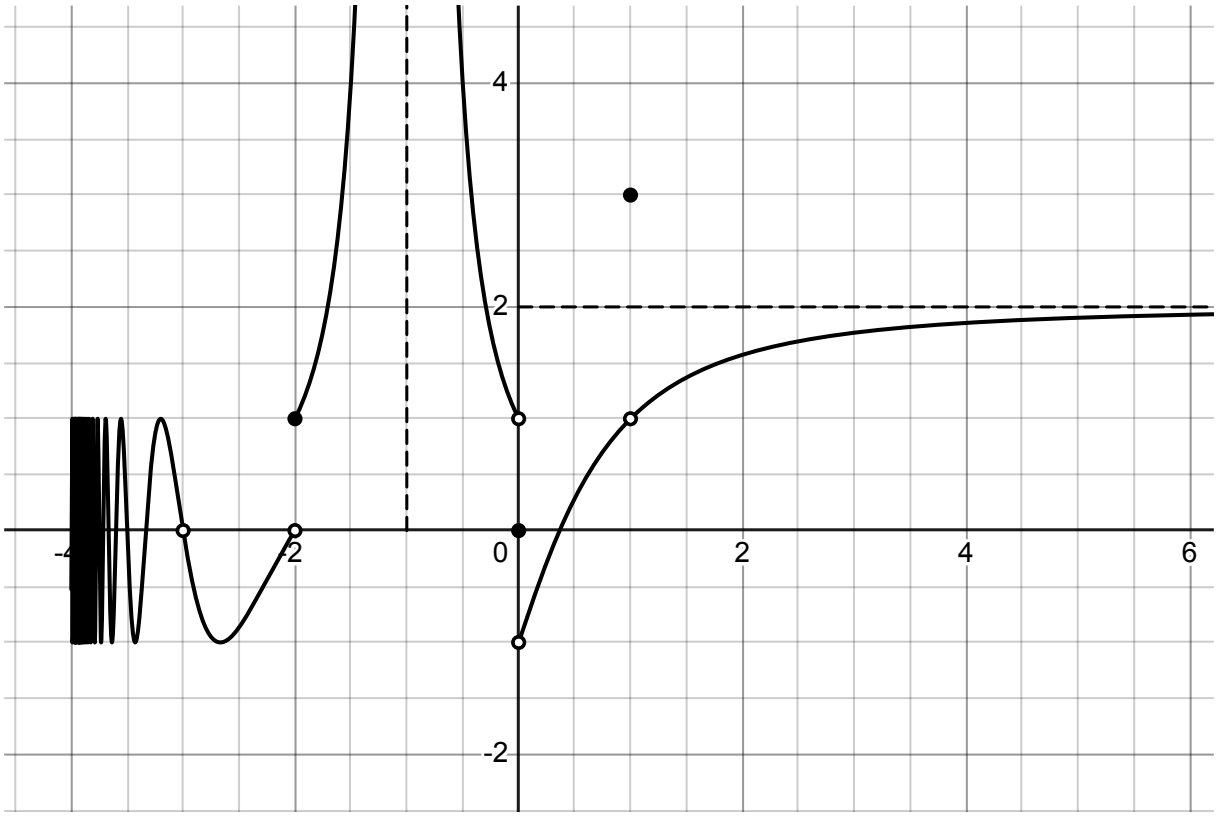

Figure 1: Graph for question 2

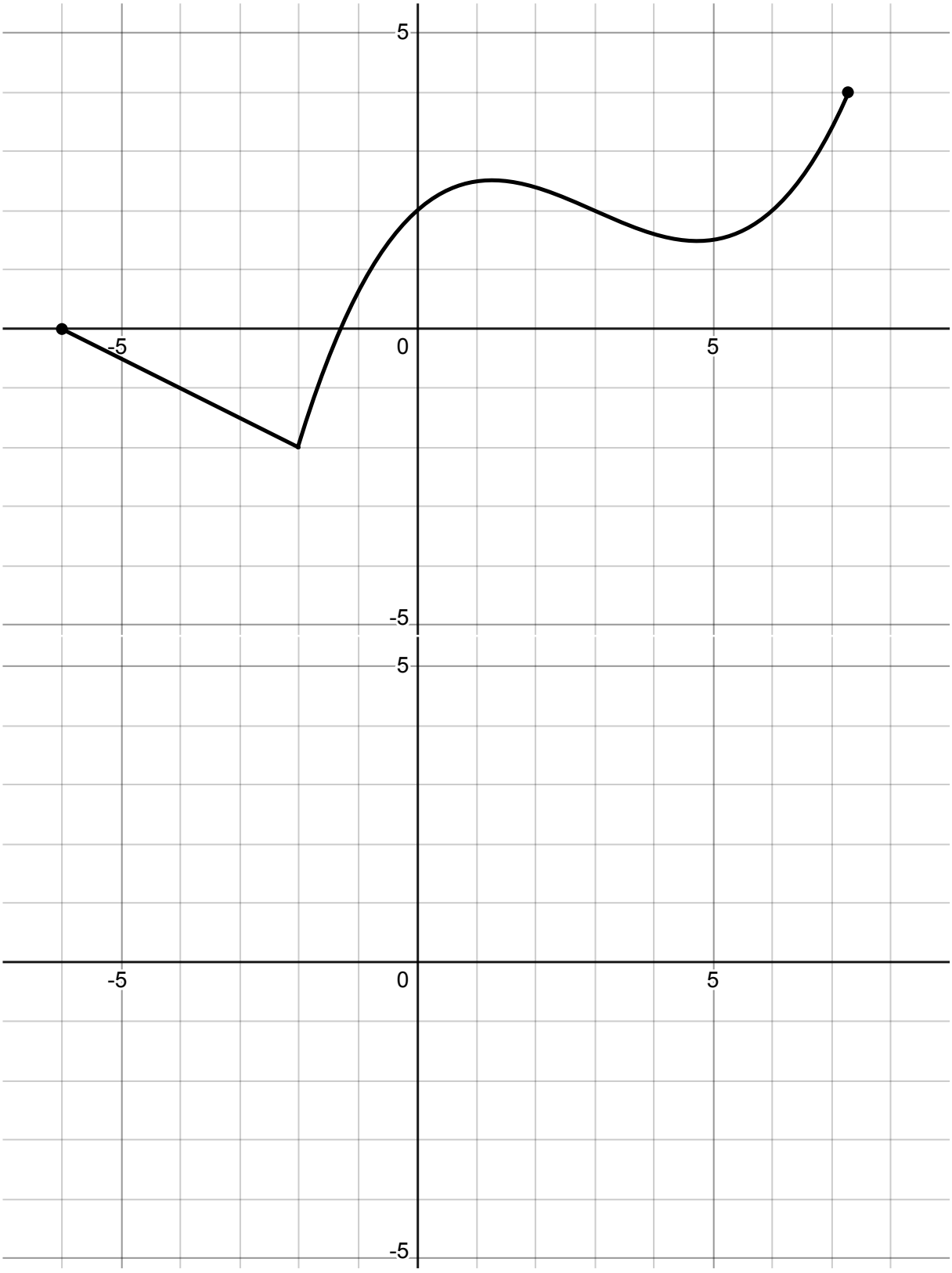

Figure 2: The graph of a function, together with an empty pair of axes for problem 9