Review for quiz 2

2025-09-25

We have our second quiz Wednesday, October 1st. Here's a first draft of a problem sheet for the quiz. It *might* be the final draft; I'm not sure.

The problems

1. Write down the definition of each of the following:
    a. Invertible Matrix Definition 3.1.1
    b. Subspace of $\mathbb{R}^n$ Definition 3.5.1
    c. Basis
        i. of $\mathbf{R}^n$ Definition 3.2.3
        ii. of a subspace Definition 3.5.4
    d. Column space of a matrix A Definition 3.5.6
    e. Null space of a matrix A Definition 3.5.10
2. For both of the matrices A and B below, find its inverse or explain briefly why you know the matrix is not invertible.
$$A = \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & a & b \\ 0 & 0 & c \\ 0 & 0 & 1 \end{bmatrix}$$
3. For what values of a is the following matrix invertible?
$$A = \begin{bmatrix} a & -1 & 0 \\ 0 & -1 & -1 \\ -1 & 0 & -1 \end{bmatrix}$$

4. The matrix A together with its reduced row echelon form R are

$$A = \begin{bmatrix} 7 & -1 & 23 & -4 & -20 \\ 2 & 6 & -6 & 1 & 7 \\ -7 & -1 & -19 & -1 & 3 \\ 0 & 8 & -16 & 1 & 11 \\ -7 & -6 & -9 & 5 & 16 \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 1 & 0 & 3 & 0 & -1 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

   a. Is A invertible?
   b. What is the dimension of the column space of A?
   c. What is the dimension of the row space of A?
   d. Find a basis for the column space of A.