

Review for exam 1

2025-09-10

Our first exam is next Wednesday, September 17! Here's our first draft of a review sheet.

Problems

1. State the following definitions:
 - a. Pivot position in a matrix. [Definition 1.4.1](#)
 - b. Linear Combination [Definition 2.1.9](#)
 - c. The span of a set of vectors [Definition 2.3.1](#)
 - d. Linear independence of a set of vectors [Definition 2.4.5](#)
 - e. Homogeneous system of equations [Top of section 2.4.3](#)
2. In this problem, we're going to consider the types of solutions that *might* occur and typically *do* occur for linear systems of various sizes.
 - a. Suppose that we have a linear system in 3 equations and 5 unknowns.
 - i. *Generally*, how many solutions do we expect there to be?
 - ii. Is it possible for there to be a *unique* solution?
 - iii. Write down a possible RREF of an augmented matrix for such a system that has no solution.
 - b. Suppose that we have a linear system in 5 equations and 3 unknowns.
 - i. *Generally*, how many solutions do we expect there to be?
 - ii. Is it possible for there to be a *unique* solution?
 - iii. Write down a possible RREF of an augmented matrix for such a system that has infinitely many solutions.
 - c. Suppose that we have a linear system in 4 equations and 4 unknowns.
 - i. *Generally*, how many solutions do we expect there to be?
 - ii. Write down a possible RREF of an augmented matrix for such a system that has exactly one solution.

3. Write down a componentwise proof of the fact that vector addition is associative. That is, if $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in $\mathbb{R}^n$, then

$$(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w}).$$

I guess you might try the same thing with commutativity.

4. Consider the vectors that form the columns of the following matrix M :

$$M = \begin{pmatrix} 0 & 1 & 1 & 2 & 2 \\ 2 & -2 & 0 & -2 & 2 \\ -1 & 1 & 0 & 3 & 1 \\ 1 & 3 & 4 & -1 & 1 \end{pmatrix}$$

- Without doing a single computation, explain why there's no way for these vectors to be linearly independent.
- Now, the RREF of M is shown below. Based on that, find a linearly independent subset of the columns whose span is the same as the span of all the columns.

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

5. Let

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \text{ and } \mathbf{z} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}.$$

Express the vector

$$\mathbf{u} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$$

as a linear combination of $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ or explain why there is no such linear combination.

6. Suppose that A is a matrix of dimensions 5×8 and B is a matrix of size 7×5 . Then, what are the dimensions of the matrices
- AB and
 - BA ?

7. Let A and B denote the matrices

$$A = \begin{pmatrix} 3 & 0 \\ 3 & -2 \\ 1 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 \\ -3 & -3 \end{pmatrix}.$$

- a. Compute AB or explain why that makes no sense.
 - b. Compute BA or explain why that makes no sense.
8. Write down 2×2 matrices that perform the following actions. In some cases, you might want to express your answer as a product of matrices that perform simpler actions.
- a. Stretches by the factor 2 in the horizontal direction and by the factor 3 in the vertical.
 - b. Stretches by the factor 2 in the horizontal direction by the factor 3 in the vertical, and also reflects across the x -axis.
 - c. Projects on the line $y = x$.
 - d. Reflects across a line through the origin that makes an angle of 17° with the x axis.