Review for quiz 1

2025-08-22

We have our first quiz next Friday, August 29th. This problem sheet represents most of the problems that will be on the quiz, though, I might add a problem after Monday's class.

Problems

1. Curious about the following limit,
$$\lim_{x\to 0}(1+x)^{3/x},$$
I used my computer to plug in several values of x that are *close* to 0 but not equal to 0. The results are shown in Table 1 below.

Table 1: Values of $f(x) = (1+x)^{3/x}$ near $x = 0$.

x	0.100000	0.010000	0.001000	0.000100	0.000010
$f(x)$	17.449402	19.788466	20.055451	20.082525	20.085236

Based on those computations, can you make a conjecture as to the approximate value of the limit? Be sure to indicate how many digits you believe to be correct and why.

2. The graph of
$$f(x) = \frac{x-1}{x^3 - x^2 + x - 1}$$
is shown in Figure 1.

   a) Judging from the figure, what do you suppose is the value of $\lim_{x\to 1} f(x)$?
   b) Use a little algebra together with the limit laws to prove that your guess is correct.

3. The Complete graph of a function f is shown in Figure 2. At each of the points $a = -1$, $a = 1$, $a = 2$ and $a = 4$, find the value of

   a) $f(a)$,

b) $\lim_{x \to a^-} f(x)$,
c) $\lim_{x \to a^+} f(x)$, and
d) $\lim_{x \to a} f(x)$.

4. Compute the following limits.

a) $\lim_{x \to 2} \dfrac{2x^2 - 3x - 2}{x - 2}$
b) $\lim_{x \to 1} \dfrac{x - 1}{x^3 + x - 2}$
c) $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$
d) $\lim_{x \to 3} \dfrac{x^2 - 6}{x - 3}$
e) $\lim_{x \to 3} \dfrac{x^2 - 6}{(x - 3)^2}$
f) $\lim_{x \to \infty} \dfrac{x^2 - 6}{x - 3}$
g) $\lim_{x \to \infty} \dfrac{x^2 - 6}{(x - 3)^2}$

Figures

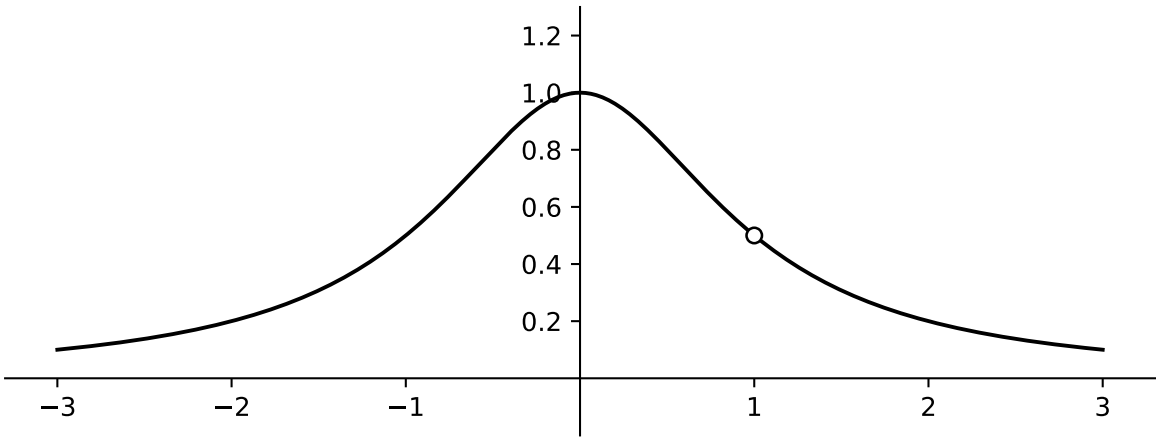

Figure 1: The graph of $f(x) = (x - 1)/(x^3 - x^2 + x - 1)$

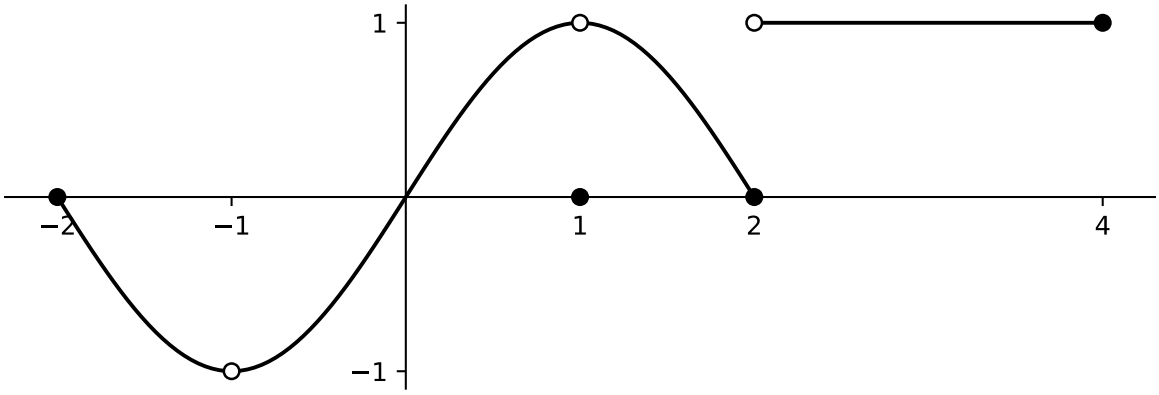

Figure 2: A graph for limits