

Review for Exam 3

We have our third exam this coming Friday, November 21st. The problems on the exam will be very much like the problems you see here. I will go over this problem sheet in class on Wednesday but it will help you immensely to think about it on your own first so *please* work it out to the best of your ability prior to meeting on Wednesday.

Problems

1. Let $f(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$; recall that this is exactly the formula for the standard normal distribution.
 - a. Sketch the graph of f and indicate the approximate locations of any inflection points.
 - b. Find $f''(x)$ and use it to find the exact locations of the inflection points.
2. The graph of $f(x) = x \ln(x)$ is shown in figure Figure 1. Find the exact location of the global minimum.
3. I need to enclose 400 square meters divided into three portions, as shown in Figure 2. I'll use the same fencing for both the exterior and interior portions. What are the dimensions of the enclosure that use the least amount of fencing?
4. Figure 3 plots the data
$$\{(-3, -2), (-2, 1), (2, -1), (3, 2)\},$$
together with its regression line of the form $f(x) = ax$.
 - a. Write down the formula for $E(a)$ representing the squared error of the regression as a function of the parameter a .
 - b. Find the value of a that minimizes that squared error.
5. Let $f(x) = x^4 - 16$.
 - a. Write down the corresponding Newton's method iteration function $N(x)$.
 - b. Take two Newton steps from the initial seed $x_0 = 1$.
 - c. Suppose that we generate a sequence via iterated applications of N . What is the limit of the resulting sequence?

6. Write down both right and left Riemann sums for

$$\int_0^2 e^x dx$$

using $n = 4$ terms. Which is an upper bound and which is a lower bound for the actual value?

7. The complete graph of a function f is shown in Figure 4. Evaluate

$$\int_{-4}^4 f(x) dx.$$

8. An object is thrown off a 90 ft tall cliff with an initial vertical velocity of 20 ft/s. Find the function $y(t)$ that describes the height of the object as a function of time t .

9. Evaluate the following definite and indefinite integrals.

a. $\int (x^2 + e^x - \sin(x) + \cos(x)) dx$

b. $\int \frac{x^3 + x^2 + x + 1}{x^2} dx$

c. $\int x \sin(x^2 + 1) dx$

d. $\int_0^2 (x^2 + 1) dx$

e. $\int_0^2 x \sqrt{x^2 + 1} dx$

10. Use u -substitution to express the following normal integrals as *standard* normal integrals:

a. $\int_0^3 \frac{1}{\sqrt{8\pi}} e^{-(x-1)^2/8} dx$

b. $\int_{-10}^{10} \frac{1}{\sqrt{50\pi}} e^{-x^2/50} dx$

Figures

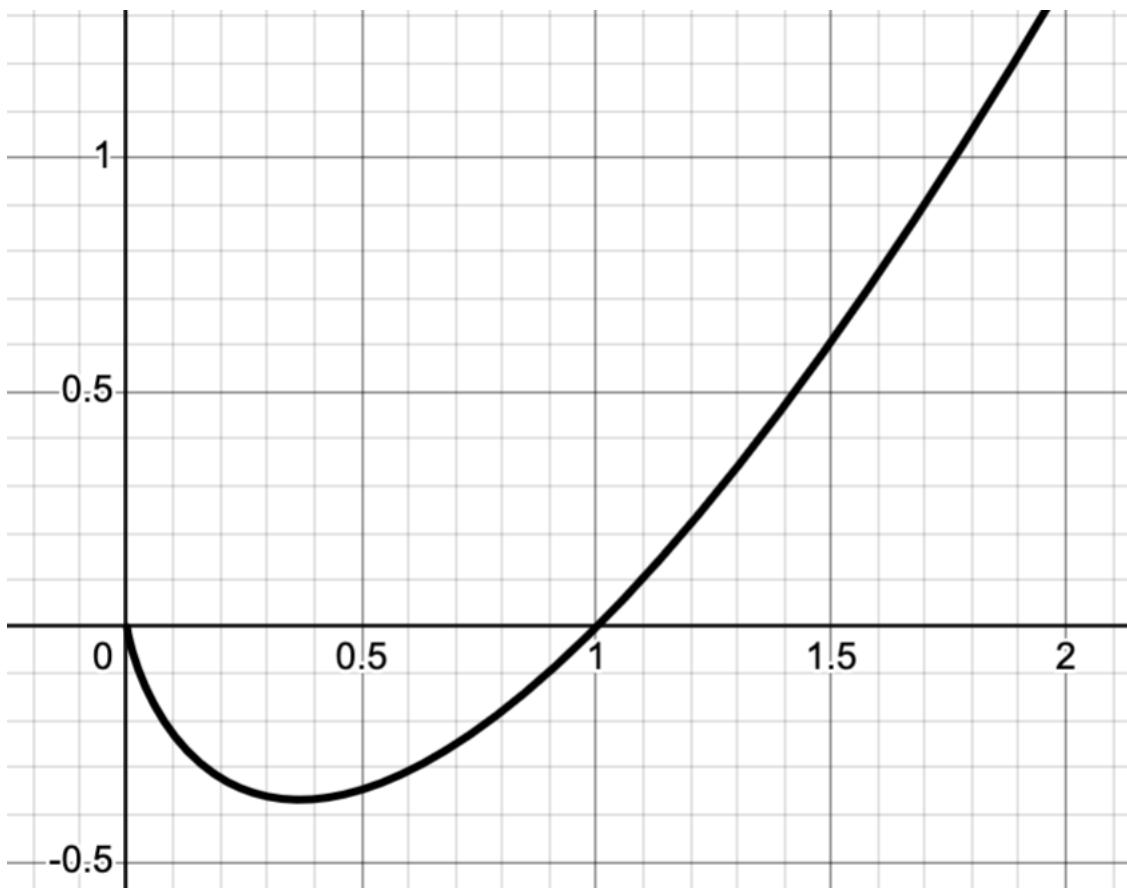


Figure 1: The graph of $f(x) = x \ln(x)$

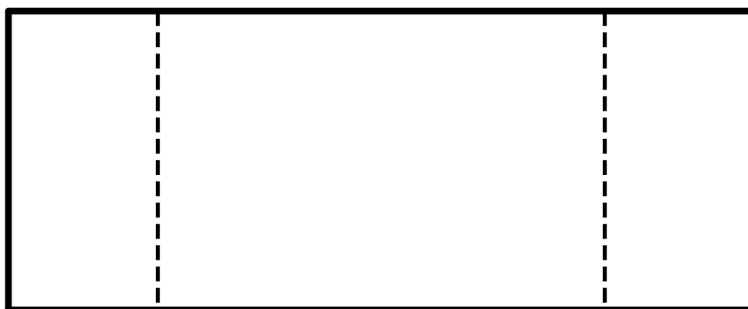


Figure 2: A partitioned corral

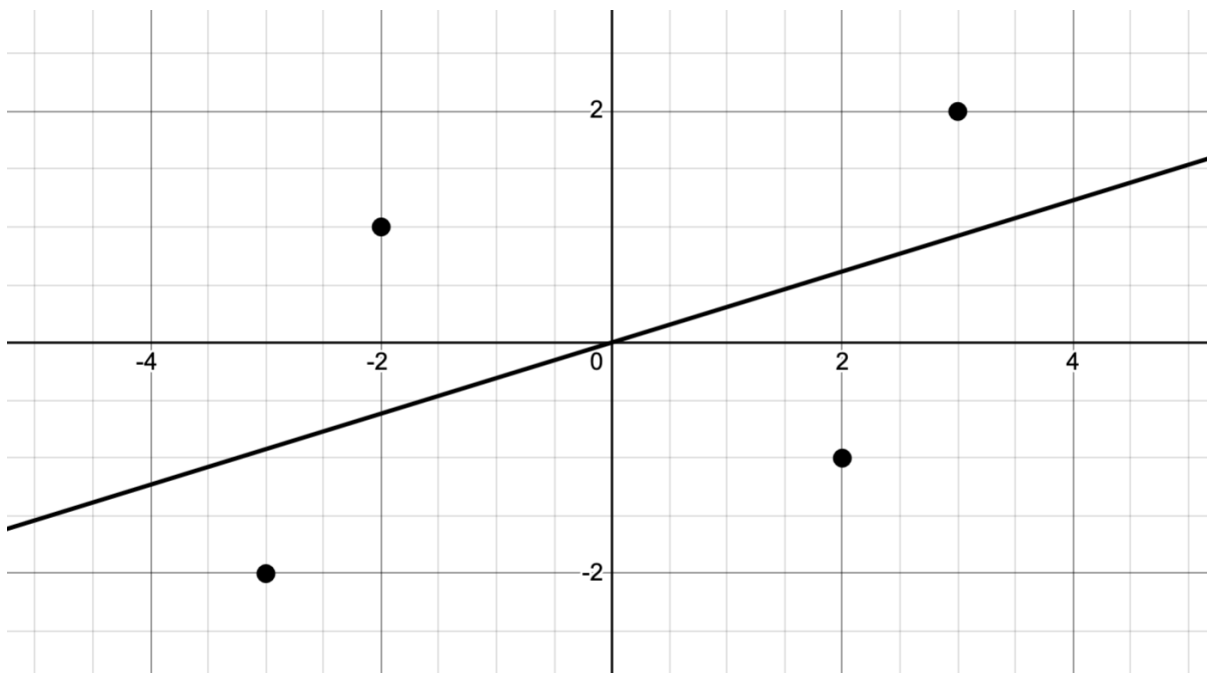


Figure 3: A little data together with its regression line

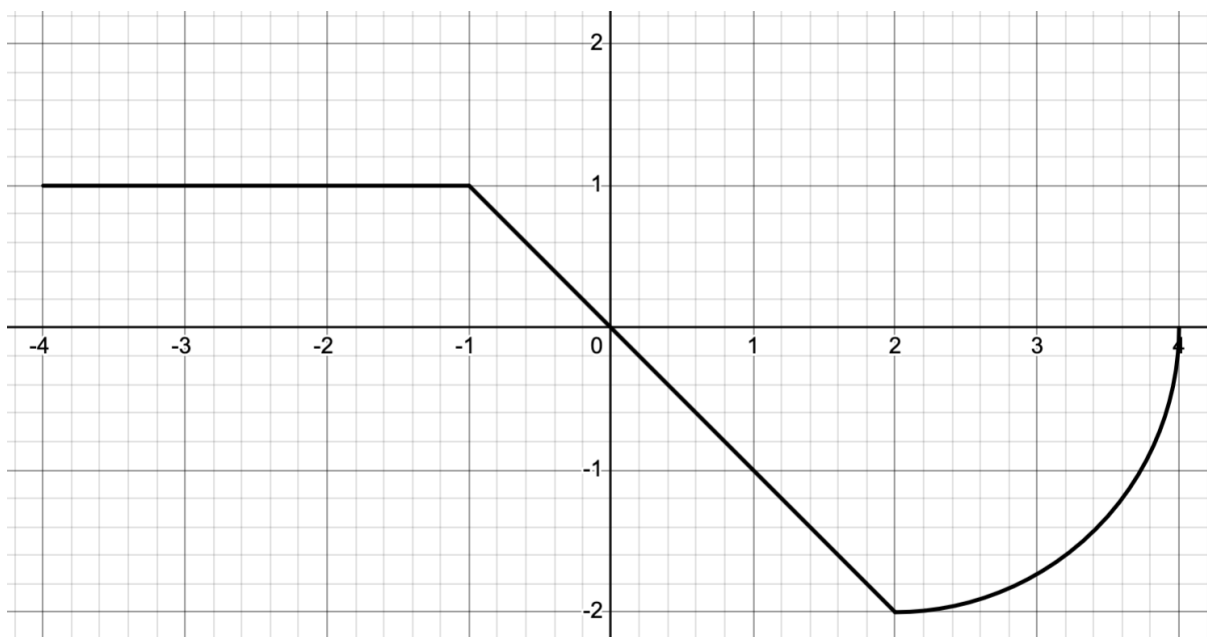


Figure 4: The complete graph of a function