Review for exam 2

We have our second exam this coming Friday, September 17th. The problems on the exam will be very much like the problems you see here. I will go over this problem sheet in class on Wednesday but it will help you immensely to think about it on your own first so *please* work it out to the best of your ability prior to meeting on Wednesday.

Problems

1. Use the differentiation rules to compute the derivatives of the following functions.

   a) $f(x) = x^2\, 2^x$
   b) $f(x) = x^3 e^{2x}$
   c) $f(x) = \dfrac{\ln(x)}{x^2}$
   d) $f(x) = (x^2+1)\sin(x)$
   e) $f(x) = e^{x^2+3x}$
   f) $f(x) = \tan(x)\cos(x^2)$
   g) $f(x) = \dfrac{x^2 e^x}{1+x^3}$
   h) $f(x) = \ln(\sin(x))$
   i) $f(x) = x^5 \tan(3x)$
   j) $f(x) = e^{\sin(x^2)}$
   k) $f(x) = \dfrac{\sqrt{x}+e^x}{x^2+1}$
   l) $f(x) = x^2 e^{2x} \sin(\pi x)\cos(x)$
   m) $f(x) = \arcsin(3x) + \arctan(x^3)$

2. Use logarithmic differentiation to evaluate the derivatives of the following functions:

   a) $f(x) = x^2 e^{2x} \sin(\pi x)\cos(x)$
   b) $f(x) = \dfrac{e^x\sqrt{x^2+1}}{\sin(x)\sqrt{x^2-1}}$

3. Suppose I invest \$10,000 at an annual rate of 5% compounded 4 times per year.

   a) How much will I have after 5 years?

b) How long will it take my money to double?
   Repeat the problem assuming your money is compounded continuously.

4. Sketch the graph of

$$f(x) = 3\sin\left(\frac{\pi}{2}x\right).$$

   Be sure to clearly indicate the x and y intercepts, as well as the maximum and minimum values.

5. Use the fact that $\lim_{\theta\to 0} \sin(\theta)/\theta = 1$ to compute

   a) $\displaystyle\lim_{x\to 0}\frac{\sin(15x)}{4x}$
   b) $\displaystyle\lim_{h\to 0}\frac{\cos(h)-1}{h}$

6. Supposing that f and g are differentiable functions, use the definition of the derivative to show that

   a. $\displaystyle\frac{d}{dx}f(x) + 3g(x) = f'(x) + 3g'(x)$
   b. $\displaystyle\frac{d}{dx}x^2 f(x) = 2x\, f(x) + x^2 f'(x)$

7. The graph of the equation $x^4 + y^2 - 3xy = 1$ is shown in Figure 1.

   a) Explain how you know for sure that the point $(1,0)$ is on the graph.
   b) Use implicit differentiation express y' as a function in terms of both x and y.
   c) Find an equation of the line that's tangent to the graph at the point $(1,0)$.

8. The graph of $f(x) = x^3 + \frac{1}{2}x - \frac{3}{2}$, together with the line $y = x$, is shown in Figure 2.

   a) Sketch the graph of of $f^{-1}(x)$ right on the same set of axes.
   b) Evaluate $(f^{-1})'(0)$.

Figures

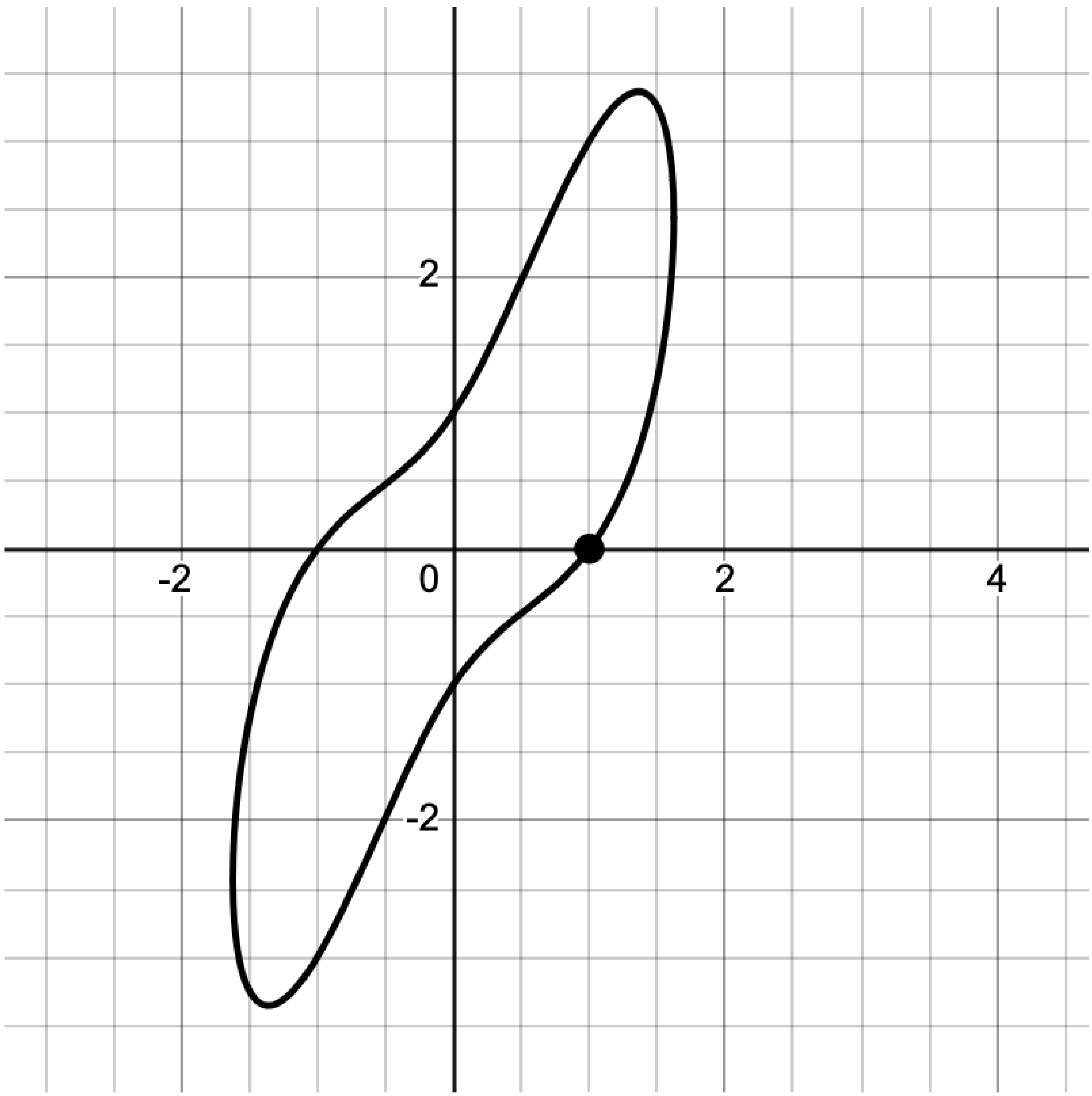

Figure 1: The graph of $x^4 + y^2 - 3xy = 1$

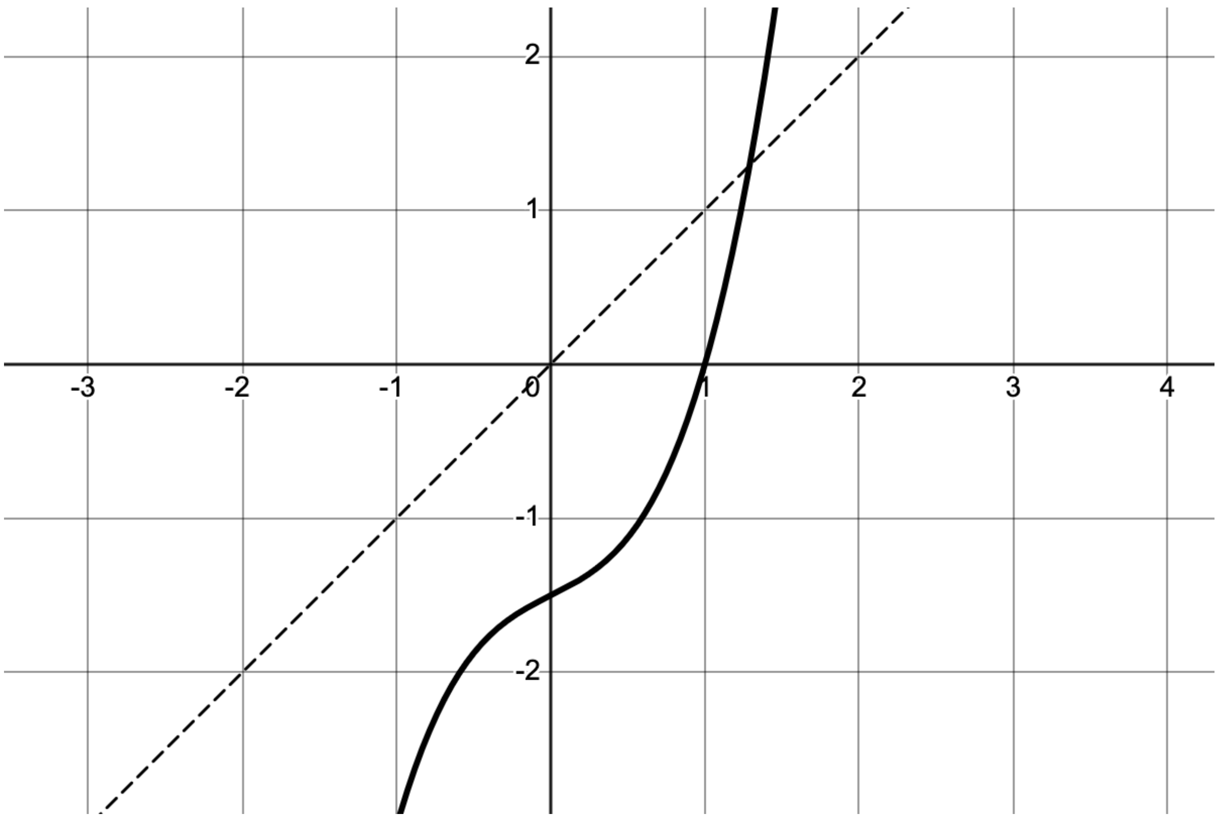

Figure 2: The graph of $f(x) = x^3 + \frac{1}{2}x - \frac{3}{2}$