Review for exam 1

We have our first exam this coming Friday, September 12th. The problems on the exam will be very much like the problems you see here. I will go over this problem sheet in class on Wednesday but it will help you immensely to think about it on your own first so *please* work it out to the best of your ability prior to meeting on Wednesday.

Problems

1. Curious about the following limit,

$$\lim_{x\to 0} \frac{\tan\left(\sqrt{x/\pi}\right)}{\sqrt{x}},$$

I used my computer to plug in several values of x that are *close* to 0 but not equal to 0. The results are shown in Table 1 below.

Table 1: Values of $f(x) = \frac{\tan(\sqrt{x/\pi})}{\sqrt{x}}$ near $x = 0$.

x	0.1000000	0.0100000	0.0010000	0.0001000	0.0000100
$f(x)$	0.5702530	0.5647890	0.5642495	0.5641956	0.5641902

Based on those computations, can you make a conjecture as to the approximate value of the limit? Be sure to indicate how many digits you believe to be correct and why.

2. The graph of a function is shown in figure Figure 1.

   a) For $a = -3, -2, -1, 0,$ and 1, find
      i) $\lim_{x\to a} f(x)$
      ii) $\lim_{x\to a^+} f(x)$
      iii) $\lim_{x\to a^-} f(x)$, and
      iv) $f(a)$

b) What are

i) $\lim_{x \to -4^-} f(x)$ and

ii) $\lim_{x \to \infty} f(x)$?

3. Let's suppose that we know that

$$\lim_{x \to 2} f(x) = 3 \text{ and } \lim_{x \to 2} g(x) = 5.$$

What can you say about the following limits?

a) $\lim_{x \to 2} (2f(x) - g(x))$?

b) $\lim_{x \to 2} \dfrac{f(x) - 3}{g(x) - 5}$?

4. Compute the following limits.

a) $\lim_{x \to 4} \dfrac{x^2 - 3x - 4}{x - 4}$

b) $\lim_{x \to -2} \dfrac{x^2 + 5x + 6}{x + 2}$

c) $\lim_{x \to 1} \dfrac{2x^2 + x + 5}{x + 1}$

Note: There's likely to be one slightly harder problem with a bit more challenging algebra. Here are a couple of examples:

d. $\lim_{x \to -2} \dfrac{x + 2}{x^3 + 3x^2 + 3x + 2}$

e. $\lim_{x \to 3/2} \dfrac{2x - 3}{2x^3 - 5x^2 + x + 3}$

5. Compute the following limits involving infinity.

a. $\lim_{x \to \infty} \dfrac{2x^2 + 3x - 2}{5x^2 - 4}$

b. $\lim_{x \to \infty} \dfrac{2x^2 + 3x - 2}{5x^3 - 4}$

c. $\lim_{x \to 2^-} \dfrac{x + 1}{x - 2}$

6. Write down a complete sentence referring to the intermediate value theorem explaining why $f(x) = 3x^7 - x - 1$ has a root between $x = 0$ and $x = 1$.

7. Find the derivatives of the following functions, *using the definition of the derivative*.

a. $f(x) = 3x^2 + 2x + 1$

b. $f(x) = x^6$

8. Find the derivatives of the following polynomials using the basic differentiation rules.

    a. $f(x) = 3x^2 + 2x + 1$
    b. $f(x) = 61x^{48} + 2x^{16} + x^{14} - x - 1$

9. The graph of the function f is shown in the top part of Figure 2. Sketch the graph of f' on the axes provided below the graph of f.

10. Let $f(x) = 4 + 4x - x^2$.

    a. Sketch the graph of f, together with the its tangent line at the point $x = 1$.
    b. Find a formula for the tangent line.

Figures

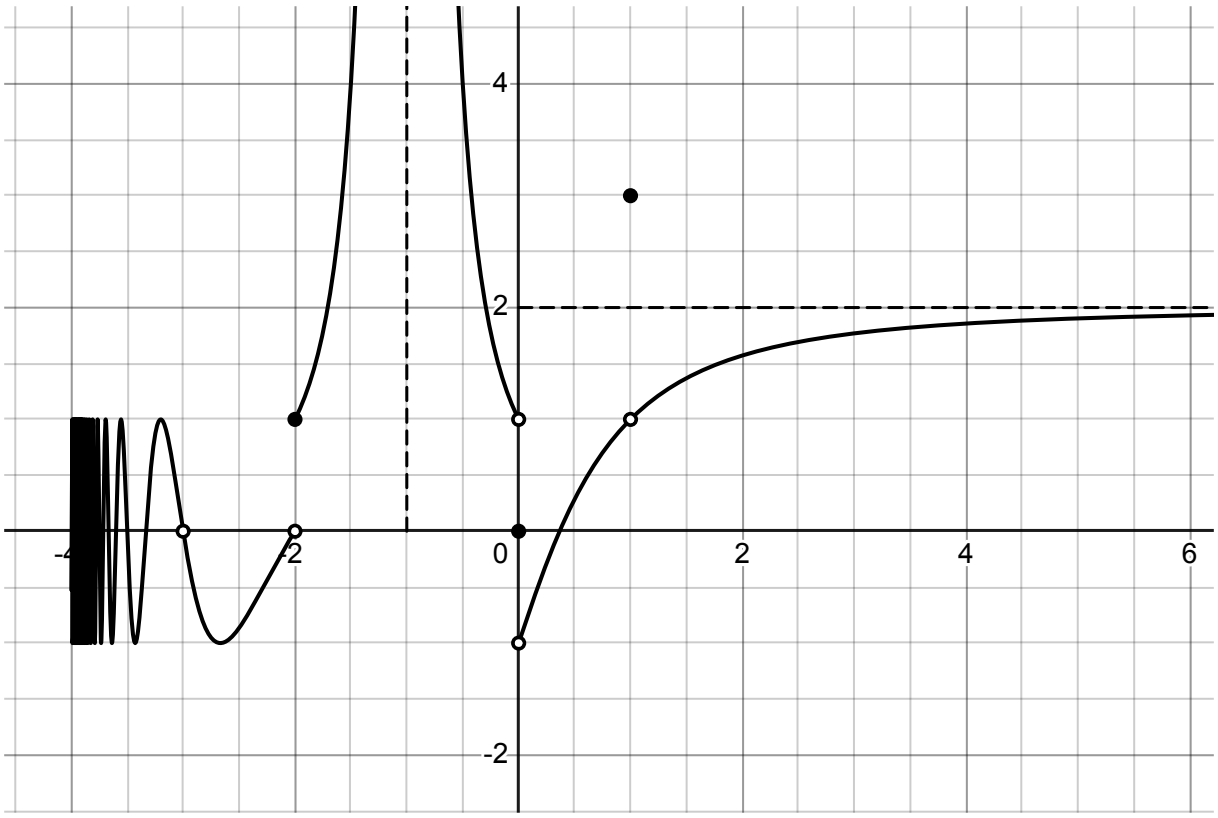

Figure 1: Graph for question 2

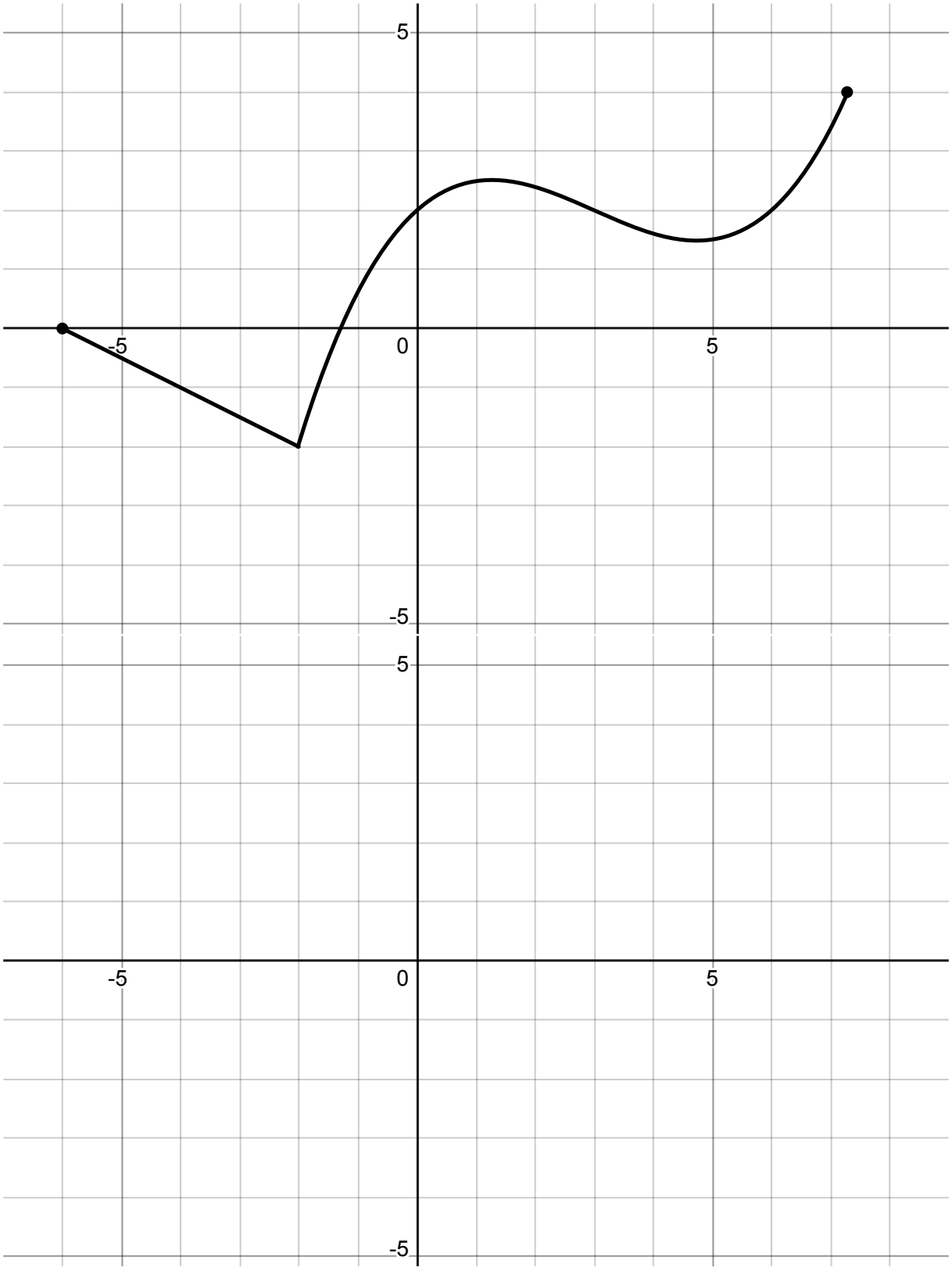

Figure 2: The graph of a function, together with an empty pair of axes for problem 9