Chaos and Fractals - Midterm Review

Our Midterm exam is next Thursday, October 5. Here are some practice problems.

1. Write down careful and complete statements of the following:
   (a) Fixed point of a function $f : \mathbb{R} \to \mathbb{R}$
   (b) Attractive fixed point of a function $f : \mathbb{R} \to \mathbb{R}$
   (c) The quadratic escape criterion for $f_c(z) = z^2 + c$
   (d) The filled Julia set and the Julia set of a quadratic for $f_c(z) = z^2 + c$
   (e) The Mandelbrot set

2. Let $f_c(x) = x^2 + c$, where $x \in \mathbb{R}$. Show that

$$x = \frac{1 - \sqrt{1 - 4c}}{2}$$

   is an attractive fixed point for f_c when $-3/4 < c < 1/4$.

3. Find and classify the fixed points of $f(z) = iz^3 + \frac{3}{2}z$.

4. Let $f_c(z) = z^2 + c$, let $g_c(z) = z^2 + 2z + c$, and let $\varphi(z) = z + 1$.
   (a) Show that $f_c \circ \varphi = \varphi \circ g_c$.
   (b) What does part (a) say about the relationship between the Julia set of f_c and the Julia set of g_c?

5. Let $f(z) = z^2 + 1$ and suppose that $z_0 \in \mathbb{C}$ satisfies $|z_0| > 2$. Show that the orbit of z_0 under iteration of f diverges to ∞.
   Note: The idea is to understand why the quadratic escape criterion is true; so no fair applying the quadratic escape criterion.

6. Use any theorem you like to establish the following statements.
   (a) The Julia set of $z^2 - 2$ is connected.
   (b) The Julia set of $z^2 - 2.1$ is totally disconnected.
   (c) The Julia set of $z^2 - 0.1$ is connected.

7. Match the points on the Mandelbrot set in figure 1 with the corresponding Julia sets.

8. Draw some good cobwebs on figure 2.

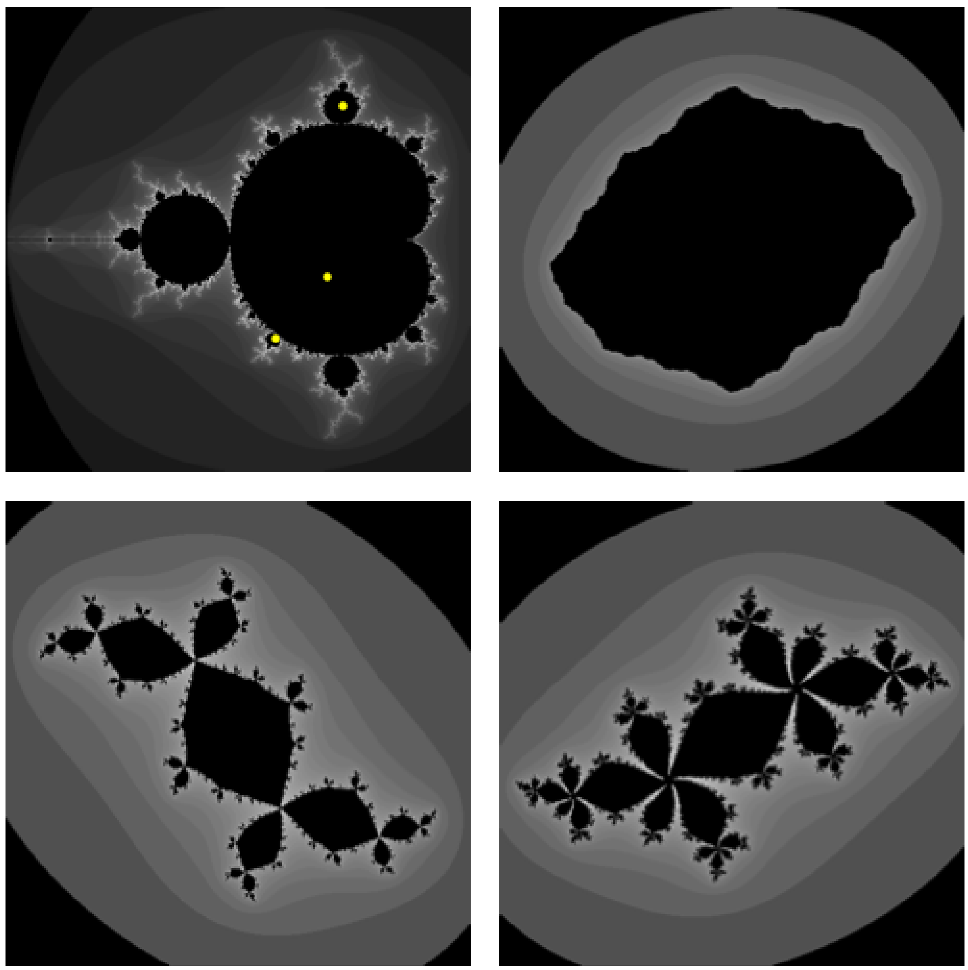

Figure 1: The Mandelbrot set with some points and corresponding Julia sets

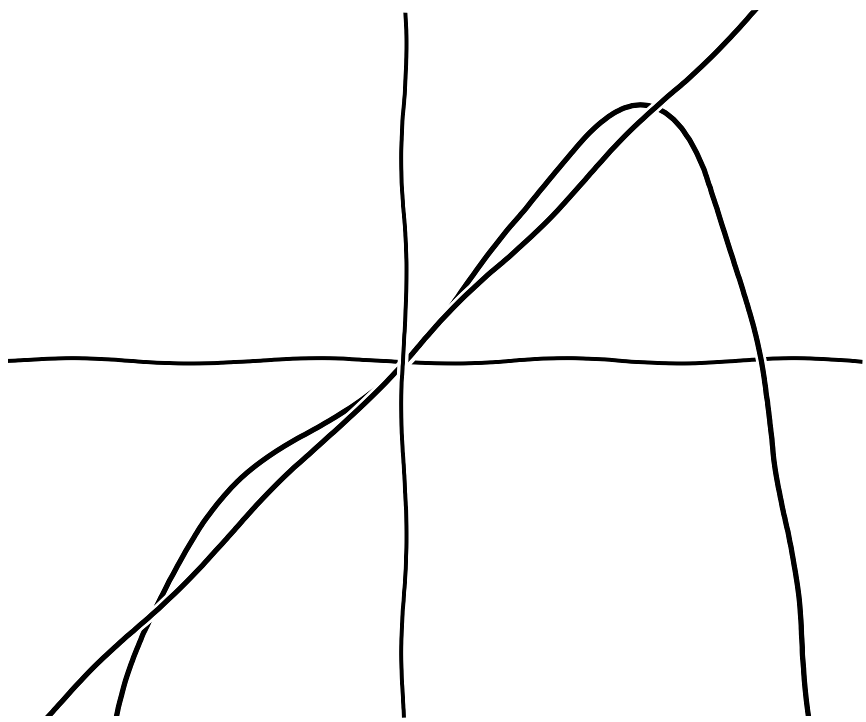

Figure 2: A function for a cobweb plot