Calc I - Review for Exam III

The third exam will be next Monday, November 20. Here are some problems that might help.

1. Let $f(x) = x^3 - 5$.
   (a) Find the corresponding Newton's method iteration function $N(x)$.
   (b) Perform two Newton iteration steps from the initial point $x_1 = 1$.
2. Suppose the volume of a sphere changes at the rate of 4 cubic centimeters per second. At what rate is the radius of the sphere increasing when it's 5 centimeters?
3. Suppose I pull the bottom of a 10 foot tall ladder away from a wall at the rate of 2 feet per second. At what rate is the top of the ladder moving towards the floor when it is 3 feet away from the floor?
4. The velocity of an object is given by $v(t) = 2t - 1$ and, at time zero, the position of the object is $p_0 = 1$. Find the position $p(t)$ of the object as a function of time.
5. Suppose I'm standing on a cliff 100 feet high and I throw an object off the cliff with a vertical velocity of 24 feet per second. Find a function $y(t)$ that indicates the objects height as a function of time and use that function to determine when the object hits the ground.
6. Evaluate the following indefinite integrals.
   (a) $\int \left(x^{12} - 8x^{10} - \sin(x) + \cos(x) - e^x\right) dx.$
   (b) $\dfrac{x^2(x-1)}{\sqrt{x^3}} dx.$
7. Use the Fundamental Theorem of Calculus to evaluate the following definite integrals
   (a) $\displaystyle\int_0^2 (3x^2 + x + 2)\, dx$
   (b) $\displaystyle\int_1^e \left(\frac{1}{x^2} + \frac{1}{x}\right) dx$
8. The complete graph of a function is shown in figure 1; it consists of two line segments and a quarter circle. Evaluate
$$\int_{-2}^{4} f(x) dx.$$
9. Write down a right Riemann with $n = 4$ terms sum to estimate
$$\int_{-1}^{1} \cos(x)\, dx.$$

10. Use Σummation notation to write down a right Riemann sum with $n = 555$ terms to estimate

$$\int_0^1 \sin(x^3)\,dx.$$

11. Find the derivative $f(x) = \arcsin(2x) + \arctan(x^3)$.

12. Figure 2 shows the graph of $f(x) = x^3 + \frac{1}{2}x - \frac{3}{2}$ together with the dashed line $y = x$.

    (a) Sketch the graph of f^{-1} on the same set of axes.

    (b) Find an equation for the line that is tangent to the graph of f^{-1} at the point where $x = 0$.

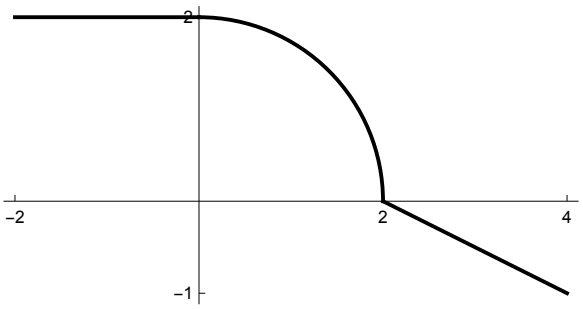

Figure 1: the graph of a function

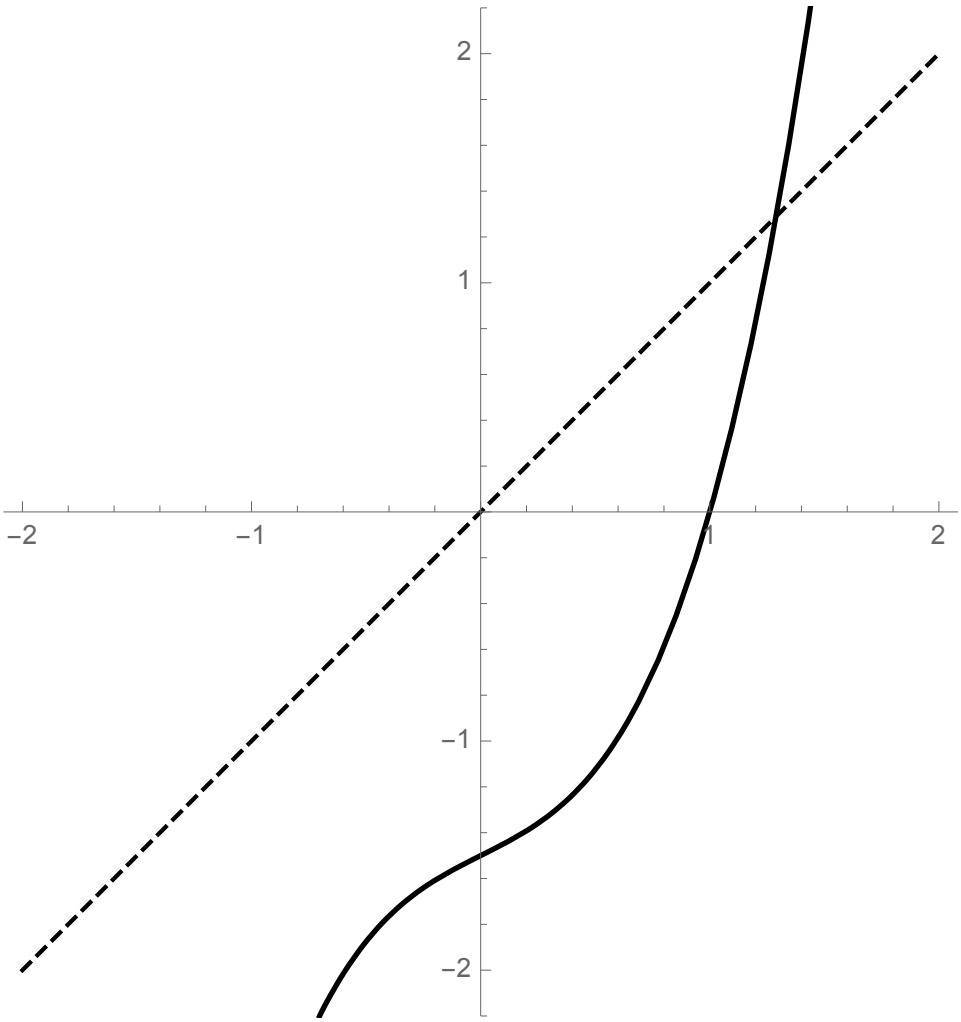

Figure 2: A function together with the line $y = x$