

Calc I - Review for Exam II

Our second exam is this Wednesday, October 25. Please work these questions out as well as you can prior to that!

1. Find the derivatives of the following functions.

(a) $f(x) = \sin(x) + \cos(x) + \ln(x)$

(b) $f(x) = \sin(x^2)$

(c) $f(x) = x^2 \ln(x^3)$

(d) $f(x) = \frac{x^2 - x - 1}{\sin^2(x)}$

(e) $f(x) = \ln\left(\frac{x\sqrt{x+1}}{(x+2)^2}\right)$

2. Use the formula for the derivatives of the sine and cosine together with the combination rules for differentiation to show that

$$\frac{d}{dx} \tan(x) = \sec^2(x).$$

3. In this problem, we're going to derive the fact that, if $f(x) = \ln(3x)$, then $f'(x) = 1/x$ using the fact that we know the inverse of f .

(a) Starting from $y = \ln(3x)$, write the equation in exponential form.

(b) Implicitly differentiate your equation from part (a) with respect to x .

(c) Solve your equation from part (b) for y' .

(d) Simplify, if necessary to show that $y' = 1/x$.

4. In this problem, we'll use the pre-drawn axes in figure 3 to draw the graphs of $f(x) = \cos(2x)$ and $f'(x) = -2\sin(2x)$ and explore their relationship.

(a) Sketch the graph of $f(x) = \cos(2x)$ on the top pair of axes.

(b) Sketch the graph of $f'(x) = -2\sin(2x)$ on the bottom pair of axes.

(c) Identify all points with horizontal tangent lines on the top graph. Projecting down, does the bottom graph cross the x axis at those points?

5. Let $f(x) = \sin(2x)$. Find an equation for the line tangent to the graph of f at the point $(\pi/8, f(\pi/8))$.

6. Let $f(x) = x^3 - 8$.
- (a) Find the corresponding Newton's method iteration function $N(x)$.
 - (b) Perform two Newton iteration steps from the initial point $x_1 = 1$.
 - (c) Suppose I take three more Newton steps. Which of the following numbers do you think I get:
 - 2.0000049116755
 - 3.0000049116755

Clearly explain your choice.

7. The complete graph of a function f is shown in figure 1
- (a) On what intervals is $f' > 0$?
 - (b) On what intervals is $f'' > 0$?
 - (c) At what points is $f' = 0$?
 - (d) At what points is $f'' = 0$?
 - (e) At what points is f discontinuous and why?
 - (f) At what points is f not differentiable and why?
8. Suppose I have 7000 feet of fence to set up a rectangular corral with three inner partitions breaking the corral into four pieces, as shown in figure 2. What is the maximum area that I can enclose?
9. Figure 4 shows the graph of

$$f(x) = 3xe^{-3x^2}.$$

Find the exact location of the maximum that you see in the picture.

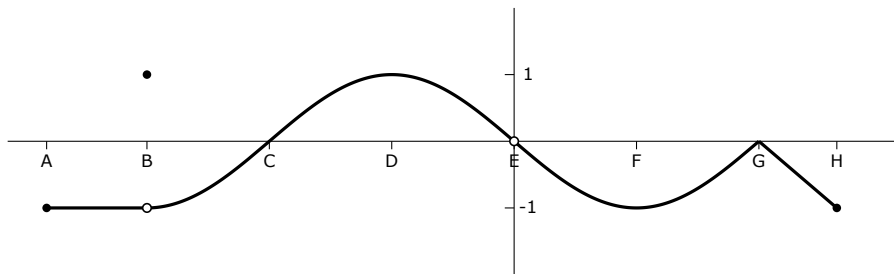


Figure 1: A pair of axes for trig functions

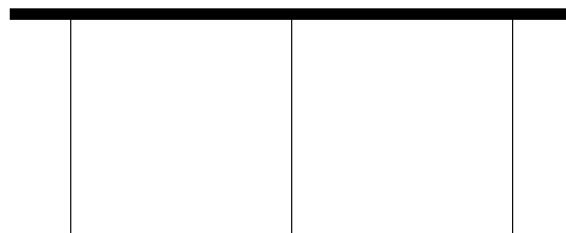


Figure 2: A partitioned corral

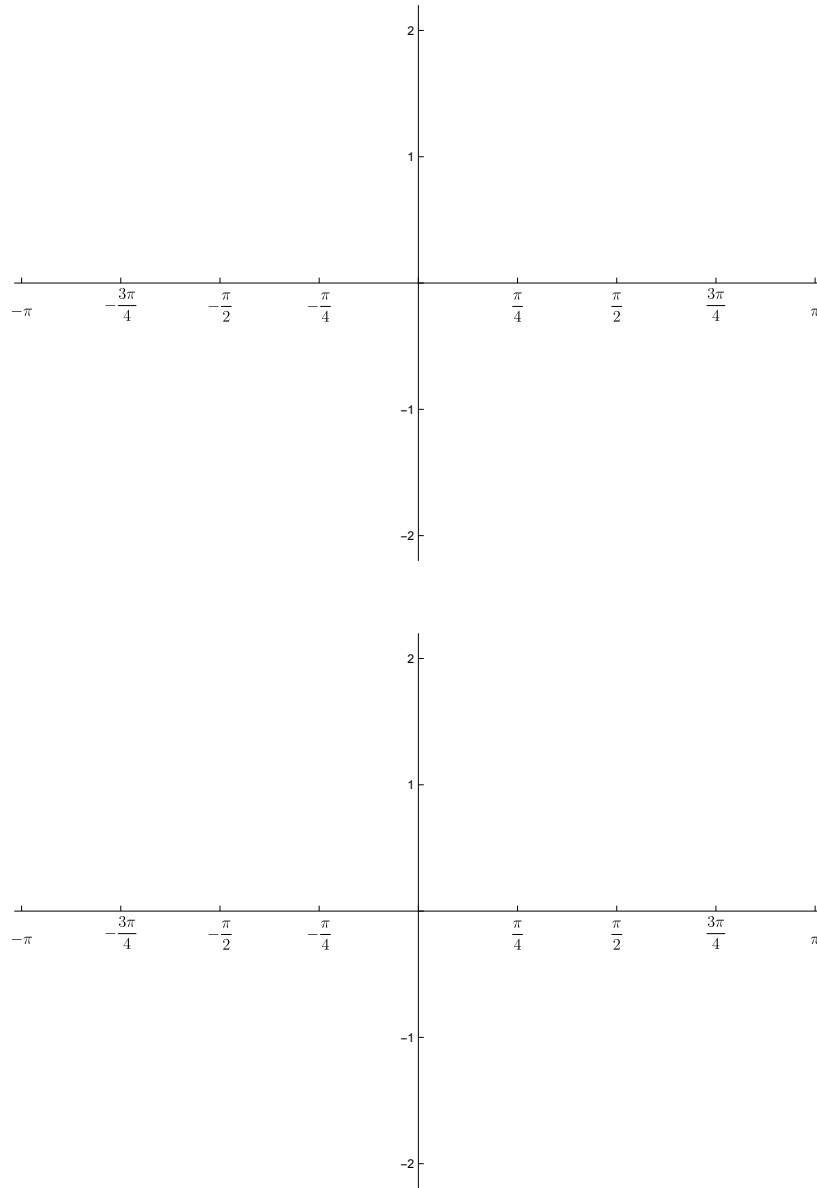


Figure 3: A pair of axes for trig functions

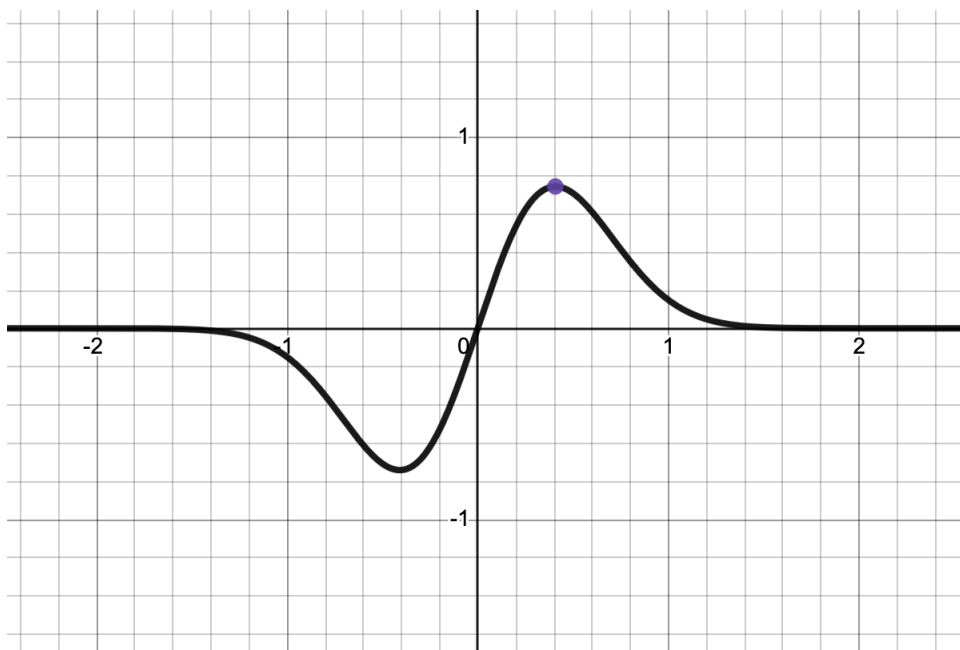


Figure 4: The graph of $f(x) = 3e^{-3x^2}$