Calc I - Review for exam I

The first exam will be this Wednesday, September 20. Here are some problems that might help.

1. Use the following steps to *estimate* the derivative of $f(x) = 6^x$.

   (a) Write down the difference quotient for f.

   (b) Use a little algebra to separate the xs from the hs.

   (c) Refer to the following table to help you find your final estimate.

t	0.1	0.01	0.001	0.0001	0.00001	0.000001
$\dfrac{6^h - 1}{h}$	1.96231	1.80791	1.79337	1.79192	1.79178	1.79176

2. Let $f(x) = x^2 - x$.

   (a) Draw the graph of f.

   (b) Write down the difference quotient for f and simplify it to the point where you can cancel the h in the denominator.

   (c) Write down the estimate for the slope of the graph of f at $x = 1$ yielded by the difference quotient using $h = 0.2$.

   (d) Write down an equation for the corresponding secant line through $(1, f(1))$ and $(1.2, f(1.2))$ and draw it on your graph.

   (e) Write down and equation for the precise tangent line at $x = 1$ and draw that on your graph as well.

3. Figure 1 shows the complete graph of a function f; its domain is $(-1, 2]$.

   (a) What are $f(1.5)$ and $\lim_{x \to 1.5} f(x)$?

   (b) What are $\lim_{x \to 1^-} f(x)$, $\lim_{x \to 1^+} f(x)$, and $\lim_{x \to 1} f(x)$?

   (c) What can you say about $\lim_{x \to -1^+} f(x)$.

4. Compute the following limits.

   (a) $\lim_{x \to 2} \dfrac{3x^2 - 7x + 2}{x - 2}$

   (b) $\lim_{x \to 3^+} \dfrac{3x^2 - 7x + 2}{x - 3}$

5. Write down a complete sentence referring to the intermediate value theorem explaining why $f(x) = 3x^7 - x - 1$ has a root between $x = 0$ and $x = 1$.

6. Find the derivatives of the following functions, *using the definition of the derivative.*

   (a) $f(x) = 2x^2 - 4x$

   (b) $f(x) = 1/\sqrt{x}$

   (c) $f(x) = x^5$

7. Find the derivatives of the following functions, using any technique you see fit.

   (a) $f(x) = 2x^2 - 4x$

   (b) $f(x) = 1/\sqrt{x}$

   (c) $f(x) = x^5$

   (d) $f(x) = x^5 \left(x^2 - x - 1\right)$

   (e) $f(x) = \left(x^2 - x - 1\right)/x$

   (f) $f(x) = 2^x + 7^x + e^x + x^e$

8. The complete graph of a function f is shown in figure 2; it consists of a line segment, a quarter-circle, and a semi-circle. Sketch the graph of f'.

9. The complete graph of a function f is shown in figure 3.

   (a) On what intervals is $f' > 0$?

   (b) On what intervals is $f'' > 0$?

   (c) At what points is $f' = 0$?

   (d) At what points is $f'' = 0$?

   (e) At what points is f discontinuous and why?

   (f) At what points is f not differentiable and why?

10. If f and g are differentiable function, prove that

$$\frac{d}{dx}(2f(x) + 3g(x)) = 2f'(x) + 3g'(x)$$

11. Use the defintion of the derivative to show that

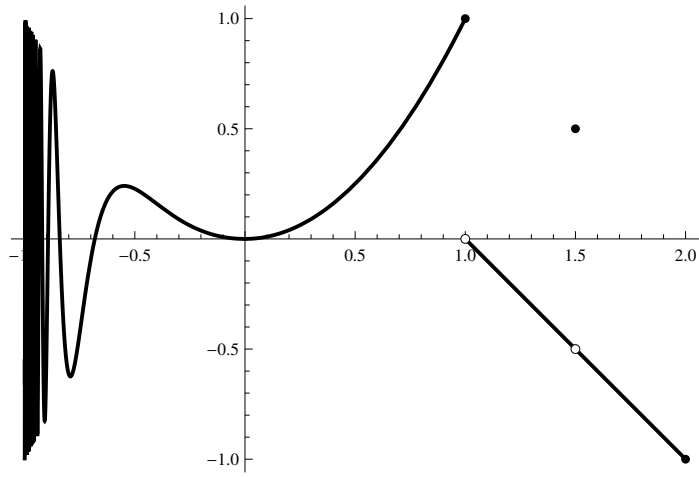$$\frac{d}{dx}\left(x^2 f(x)\right) = 2x f(x) + x^2 f'(x).$$
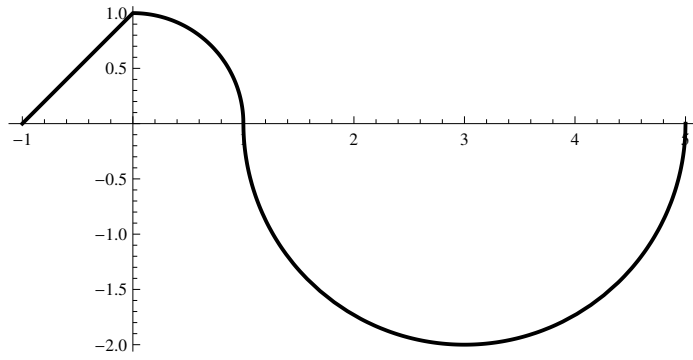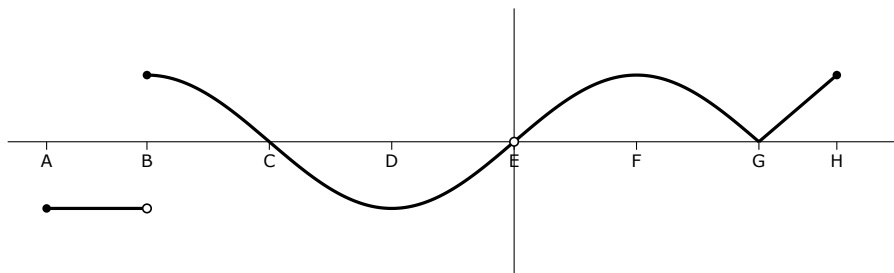
Figure 1: The graph for problem 4

Figure 2: The graph for problem 8

Figure 3: The graph for problem 9