Calc I - Final Review

Our final exam is next Friday, December 8. The problem sheet serves as a review sheet for that final. It consists mostly of problems off of past exams, though there are a few "Bonus" problems that are variations that you should know. In addition, there is a section devoted entirely to u-substitution.

Exam I

3. Compute the following limits.

   (a) $\lim_{x \to 2^+} \dfrac{x+1}{x-2}$

   (b) $\lim_{x \to 3} \dfrac{x-3}{3x^2 - 10x + 3}$

Bonus: $\lim_{\theta \to 0} \dfrac{\sin(3\theta)}{\theta}$

4. Write down a complete sentence referring to the intermediate value theorem explaining why $f(x) = 3x^3 - x - 1$ has a root between $x = 0$ and $x = 1$.

5. Find the derivatives of the following functions, using the differentiation rules we've learned in class.

   (a) $f(x) = x^2 - x$

   (b) $f(x) = \frac{1}{x}$

   (c) $f(x) = \sqrt{x}$

   (d) $f(x) = x^5(e^x + x + 1)$

   (e) $f(x) = \frac{e^x - 1}{x^2}$

6. Find the derivatives of the following functions, *using the definition of the derivative.*

   (a) $f(x) = x^2 - x$

   (b) $f(x) = \frac{1}{x}$

Bonus: $f(x) = \sqrt{x}$

8. The complete graph of a function f is shown in figure 1. Sketch the graph of f' on the axes provided below the graph of f.

Exam II

2. Find the derivatives of the following functions.

   (a) $f(x) = \sin(x) + \cos(x) + \ln(x)$

   (b) $f(x) = e^{-x^2}$

   (c) $f(x) = x^3 \ln(x^2)$

   (d) $f(x) = \dfrac{\cos^2(x)}{x^2+1}$

   (e) $f(x) = \ln\left(\dfrac{\sqrt{x^2+1}}{(x+2)^2}\right)$

4. Let $f(x) = x^2 - 9$.

   (a) Find the corresponding Newton's method iteration function $N(x)$.

   (b) Perform two Newton iteration steps from the initial point $x_1 = 1$.

   (c) Suppose I take three more Newton steps. Which of the following numbers do you think I get:

      - 2.000000001396984
      - 3.000000001396984

   Clearly explain your choice.

6. I need to build a corral partitioned into two pieces against an existing wall as shown in figure 2. The total area enclosed needs to be 500 square feet. The material for the outer portion costs 10 dollars per foot while the material for the inner portion costs 5 dollars per foot.

   (a) What are the dimensions of the cheapest possible corral?

   (b) How much does the cheapest possible corral cost?

7. Figure 3 shows the graph of

$$f(x) = 3xe^{-2x/5}.$$

   Find the exact location of the maximum that you see in the picture.

Exam III

1. Suppose the volume of a sphere changes at the rate of 3 cubic centimeters per second. At what rate is the radius of the sphere increasing when it's 7 centimeters?

Bonus: Suppose I pull the bottom of a 10 foot tall ladder away from a wall at the rate of 2 feet per second. At what rate is the top of the ladder moving towards the floor when it is 3 feet away from the floor?

2. Suppose I'm standing on a cliff 240 feet high and I throw an object off the cliff with a vertical velocity of 32 feet per second. Find a function $y(t)$ that indicates the objects height as a function of time and use that function to determine when the object hits the ground.

3. Evaluate the following indefinite integrals.

   (a) $\displaystyle\int \left(x^{12} + \frac{2}{x} - \sin(x) + \cos(x) - e^x \right) dx.$

   (b) $\displaystyle\int \left(\frac{1}{1+x^2} - \frac{1}{\sqrt{1-x^2}} \right) dx.$

   (c) $\displaystyle\int \frac{x^2(x^2-1)}{x^3} dx.$

4. Use the Fundamental Theorem of Calculus to evaluate the following definite integrals

   (a) $\displaystyle\int_0^2 (x^2-1)\, dx$

   (b) $\displaystyle\int_1^{e^2} \frac{1}{x}\, dx$

5. The complete graph of a function is shown in below; it consists of three line segments and a quarter circle. Evaluate

$$\int_{-2}^{4} f(x)dx.$$

6. Use summation notation to write down a right Riemann with $n = 100$ terms sum to estimate

$$\int_0^4 \sin(x^2)\, dx.$$

u-Substitution

1. Use u-substitution to evaluate the following indefinite integrals.

   (a) $\displaystyle\int x\sqrt{x^2+1}\, dx$

   (b) $\displaystyle\int \sqrt{2x+1}\, dx$

   (c) $\displaystyle\int \sin^3(x)\cos(x)\, dx$

   (d) $\displaystyle\int \frac{1}{x\ln(x)}\, dx$

2. Use u-substitution to evaluate the following definite integrals.

   (a) $\displaystyle\int_1^2 x^2(x^3+1)^9\, dx$

   (b) $\displaystyle\int_{-1}^{1} xe^{\sin(x^2)}\, dx$

Figures

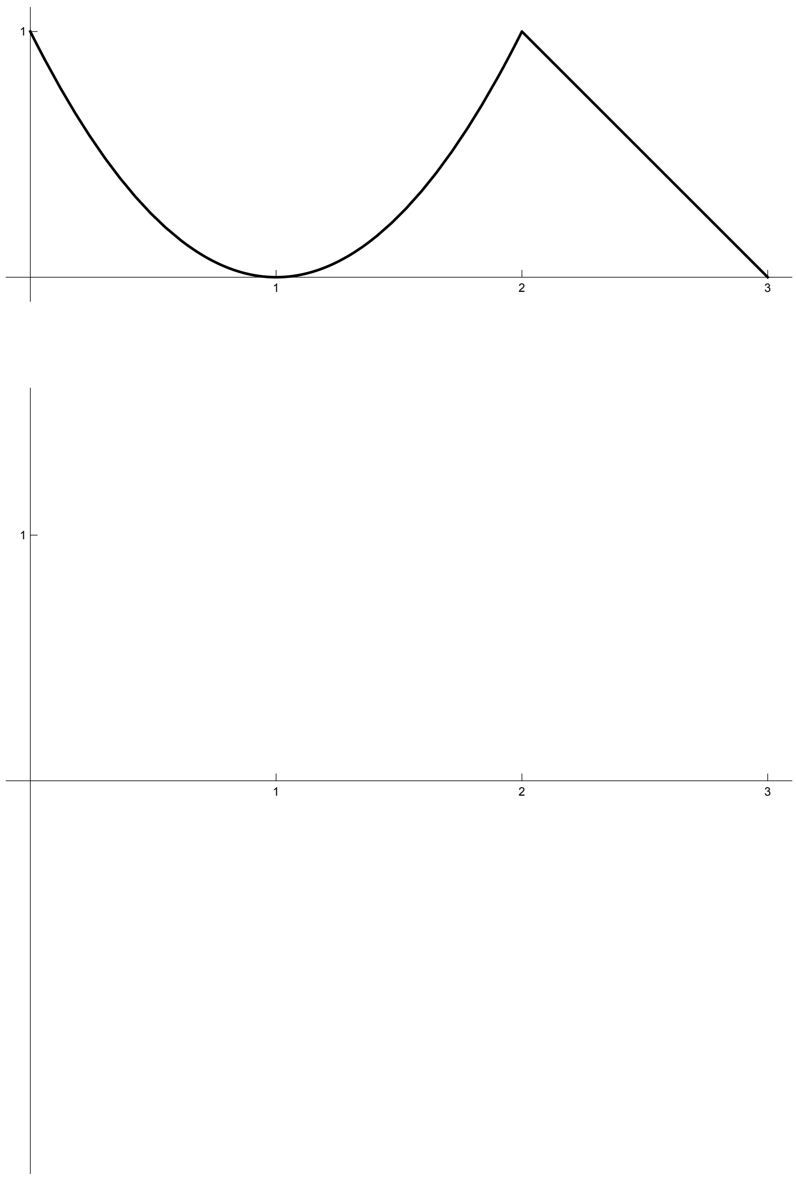

Figure 1: The graph of a function with a spare set of axes

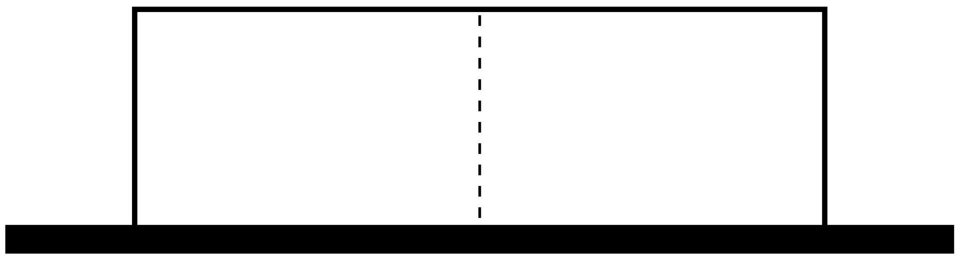

Figure 2: A corral

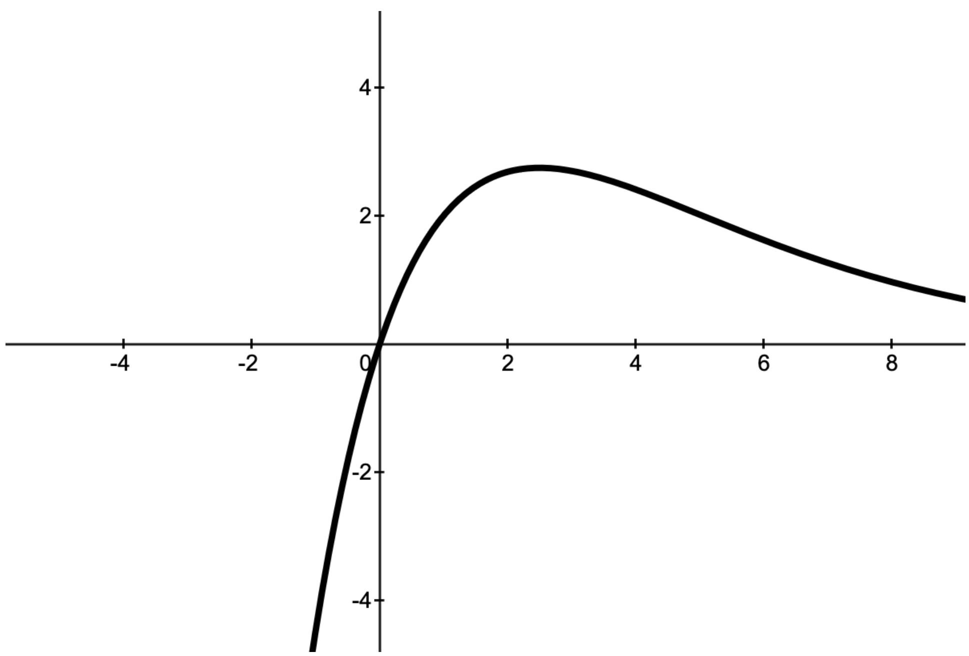

Figure 3: The graph of a function to maximize

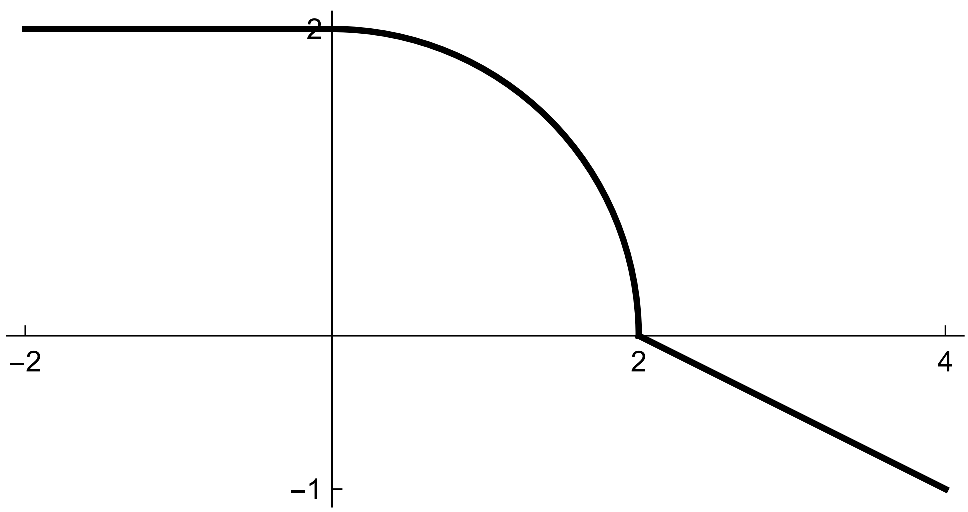

Figure 4: The complete graph of a function