Calc III - Review 2

1. Find *both* partial derivatives of the following functions.
   (a) $f(x, y) = xe^{xy}$
   (b) $f(x, y) = y\ln(x - xy)$
   (c) $f(x, y) = \dfrac{x^2 - y}{y^2 - x}$
2. Let $f(x, y) = xy^3$.
   (a) Compute ∇f.
   (b) From the point $(2, 1)$, in what direction $\vec{u}$ is $f$changing the fastest?
   (c) From the point $(2, 1)$, is there any direction $\vec{u}$ so that $D_{\vec{u}}f(2, 1) = 10$?
3. Find all points on the curve $x^2 + y^2 - xy = 4$ where a normal vector to the curve is perpendicular to the vector $\langle 1, 2\rangle$.
4. Find and classify the critical points of the function $f(x, y) = 8x^2 + 2xy^2 + 4y^2$.
5. Find and classify the critical points of the function $f(x, y) = x^3 - 6xy - y^2$.
6. Find the equation of the plane tangent to the graph of $2x^2 - y^2 + z^2 = 8$ at the point $(2, 1, 1)$.
7. Let $f(x, y) = 3(x - 2)^2 + (y + 1)^2$.
   (a) Sketch and label several contours of f.
   (b) What does part (a) tell you about local maximum and minimum values of f?
   (c) Sketch the gradient field of f over your contour diagram. Pay special attention to the direction an relative magnitudes of the gradient vectors.
8. Use the method of Lagrange multipliers to find the extremes of $f(x, y) = 2x+4y$ subject to $x^2+y^2 = 20$.
9. Let $f(x, y, z) = y^2z - x^2y$.
   (a) Compute ∇f.
   (b) Find an equation of the plane tangent to the surface $f(x, y, z) = 10$ at the point $(1, 2, 3)$.
10. Figure 1 (a) shows the contour diagram of $f(x, y) = xye^{-(x^2+y^2)}$.
    (a) On the figure, identify any maximum, minimum, or saddle points.
    (b) Find the exact location of all maxima.
11. Figure 2 (b) shows the contour diagram of $f(x, y) = xy(1 - x - y)$.
    (a) On the figure, identify any maximum, minimum, or saddle points.
    (b) Find the exact location of all saddle points.

12. The contour plot of a function is shown in figure 3.

    (a) For each point that I've marked on the figure, draw the corresponding gradient vector emanating from that point. Be sure to pay special attention to both the direction and relative magnitude of your gradient vectors.

    (b) Label any max, min, or saddle points that you see.

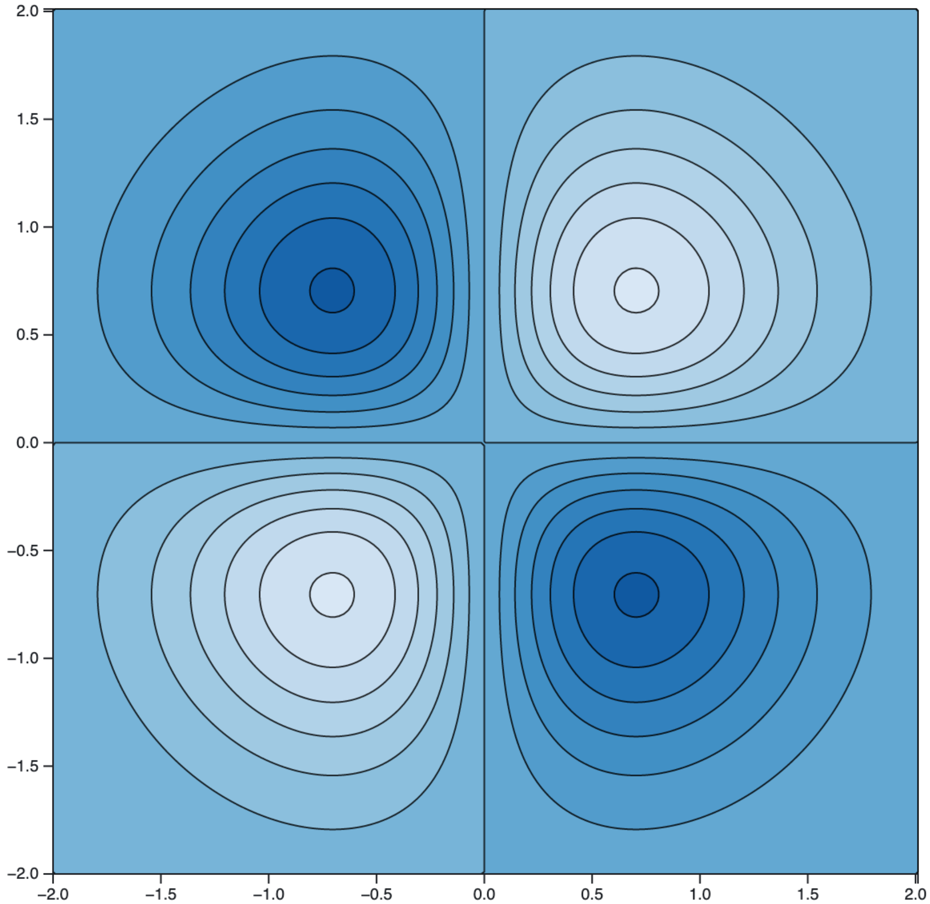

Figure 1: The contour diagram of $f(x,y) = xye^{-\left(x^2+y^2\right)}$.

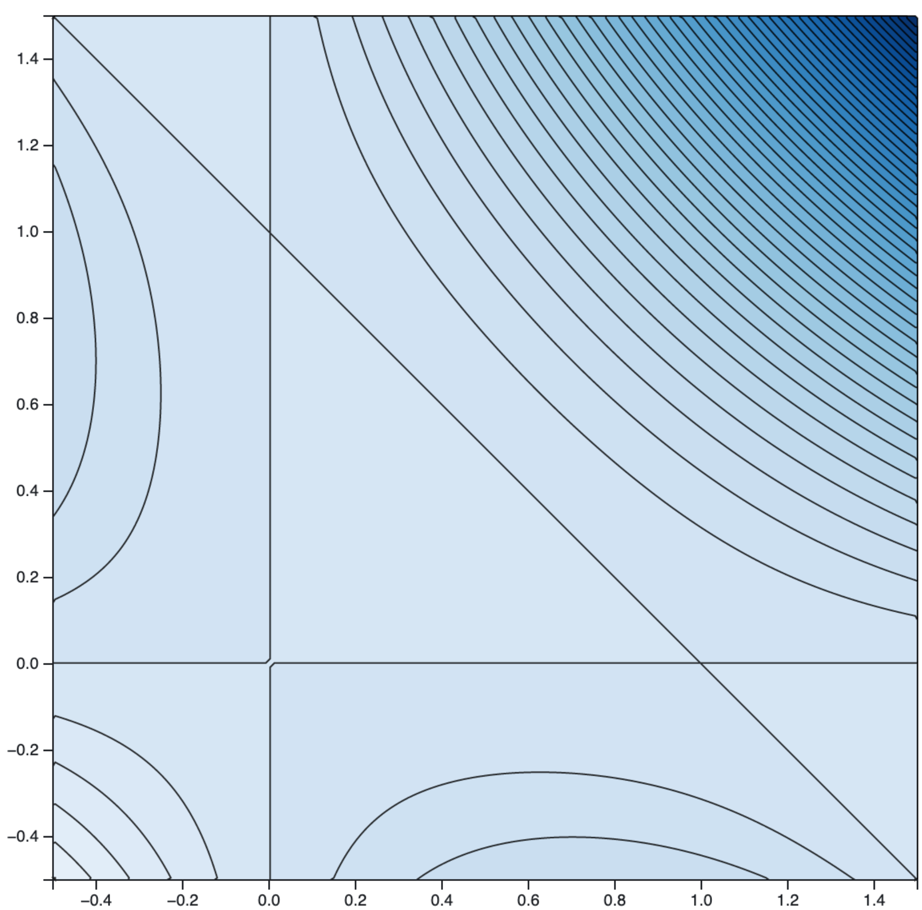

Figure 2: The contour diagram of $f(x, y) = xy(1 - x - y)$

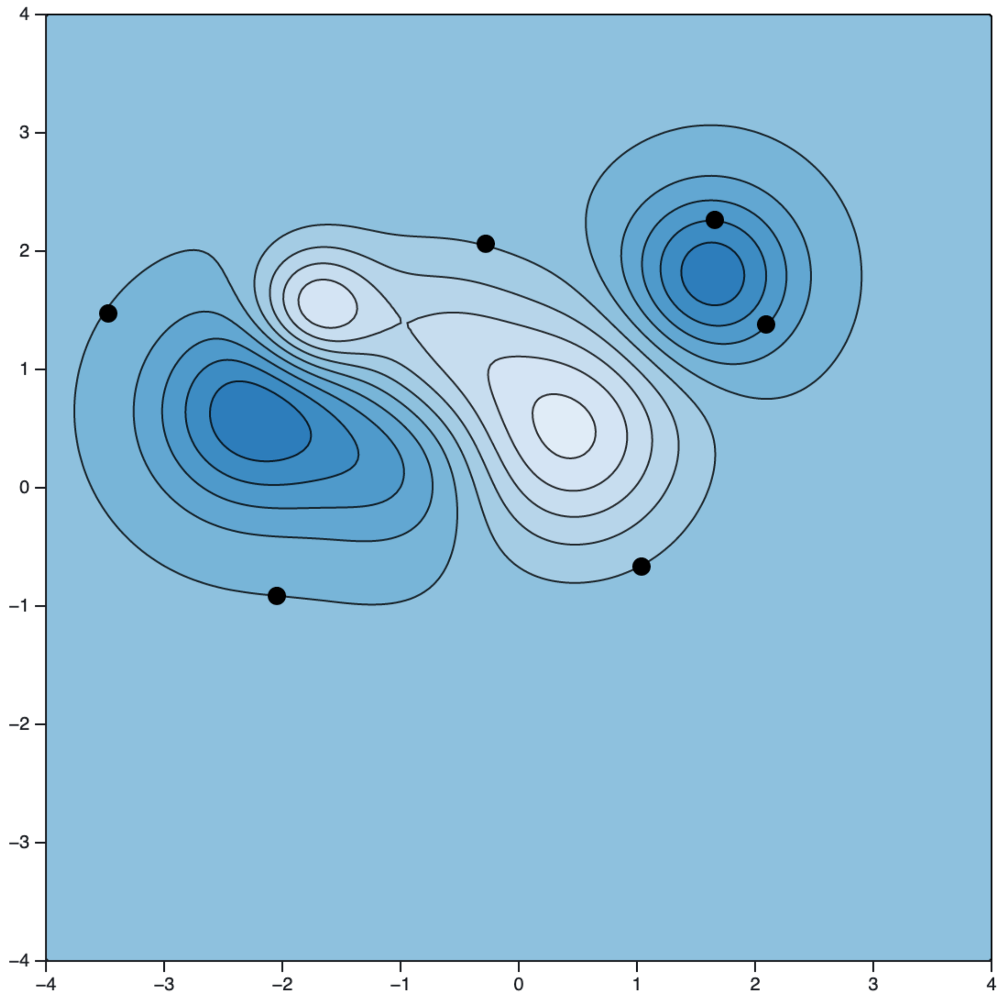

Figure 3: A contour plot with points