

# Calc III - Review I

The first exam will be next Friday, September 16. We will spend some time discussing a few of these problems in class on Wednesday but you should work them all out to the best of your ability prior to that. Understanding the problems on this sheet will help you greatly on the exam.

If you think that one of these problems has a mistake, you should try to find the mistake and fix it.

1. Sketch the curve  $y^2 + 4z^2 = 4$  in the  $yz$ -plane. What does the graph of this equation look like in  $xyz$ -space?
2. The equation  $x^2 + y^2 + z^2 = 2z$  describes a sphere. What are the center and radius of that sphere?
3. An object moves according to the parametrization  $\vec{p}(t) = \langle \sin(12t), -t + \cos(12t) \rangle$ . Describe the motion determined by  $\vec{p}(t)$ .
4. Possible proofs
  - (a) Let  $\mathbf{v} = \langle x, y \rangle$  and let  $\mathbf{w} = r\mathbf{v}$ . Show that  $\|\mathbf{w}\| = r\|\mathbf{v}\|$ .
  - (b) Prove that the two-dimensional dot product is commutative and distributive over vector addition. Is it associative? Why?
  - (c) Let  $\vec{a} = \langle a_1, a_2, a_3 \rangle$  and  $\vec{b} = \langle b_1, b_2, b_3 \rangle$  be three dimensional vectors. Prove that  $\vec{a} \times \vec{b}$  is perpendicular to  $\vec{a}$ .
5. Let  $\vec{u} = \langle 3, 1, -1 \rangle$  and  $\vec{v} = \langle 1, -3, -2 \rangle$ .
  - (a) Compute  $\text{proj}_{\vec{v}}\vec{u}$ , the vector projection of  $\vec{u}$  onto  $\vec{v}$ .
  - (b) What is the exact angle between  $\vec{u}$  and  $\vec{v}$ ?
6. Find an equation of the plane that is tangent to the sphere  $x^2 + y^2 + z^2 = 14$  at the point  $(1, 2, 3)$ .
7. Find the equation of a plane containing the points  $(3, 3, 3)$ ,  $(0, 2, 2)$ , and  $(-3, 0, 3)$  or explain why no such plane exists.
8. Let  $p(t) = (1 + t, -2t, 1 - t)$  and  $q(t) = (1 - t, 2t, 1 + 2t)$  be the parameterizations of two lines.
  - (a) Find the point of intersection of the two lines.
  - (b) Find an equation of the plane that contains the two lines.
9. Let  $p(t) = (1 + 2t, 1 + t, 1 - t)$  and  $q(t) = (4 + 3t, 3 + 2t, 1 + t)$  be the parameterizations of two lines. Find the distance between those lines.
10. Let  $f(x, y) = 2x^2 - y^2 + x$ . Find an equation of the plane tangent to the graph of  $f$  at the point  $(1, 2)$ .
11. Let  $f(x, y) = x \sin(xy^2)$ . Find  $\partial f / \partial x$  and  $\partial f / \partial y$ .

12. Match the groovy function below with the groovy picture in figure 1.

- (a)  $\vec{p}(t) = \langle \cos(3t), \sin(3t) \rangle$ .
- (b)  $\vec{p}(t) = \langle \cos(3t), 0, \sin(3t) \rangle$ .
- (c)  $\vec{p}(t) = \langle \cos(3t), t, \sin(3t) \rangle$ .
- (d)  $\vec{p}(t) = \langle \cos(3t) + t, \sin(3t) \rangle$ .
- (e)  $f(x, y) = \sin(x^2 - y^2)$
- (f)  $f(x, y) = e^{-(x^2+2y^2)}$

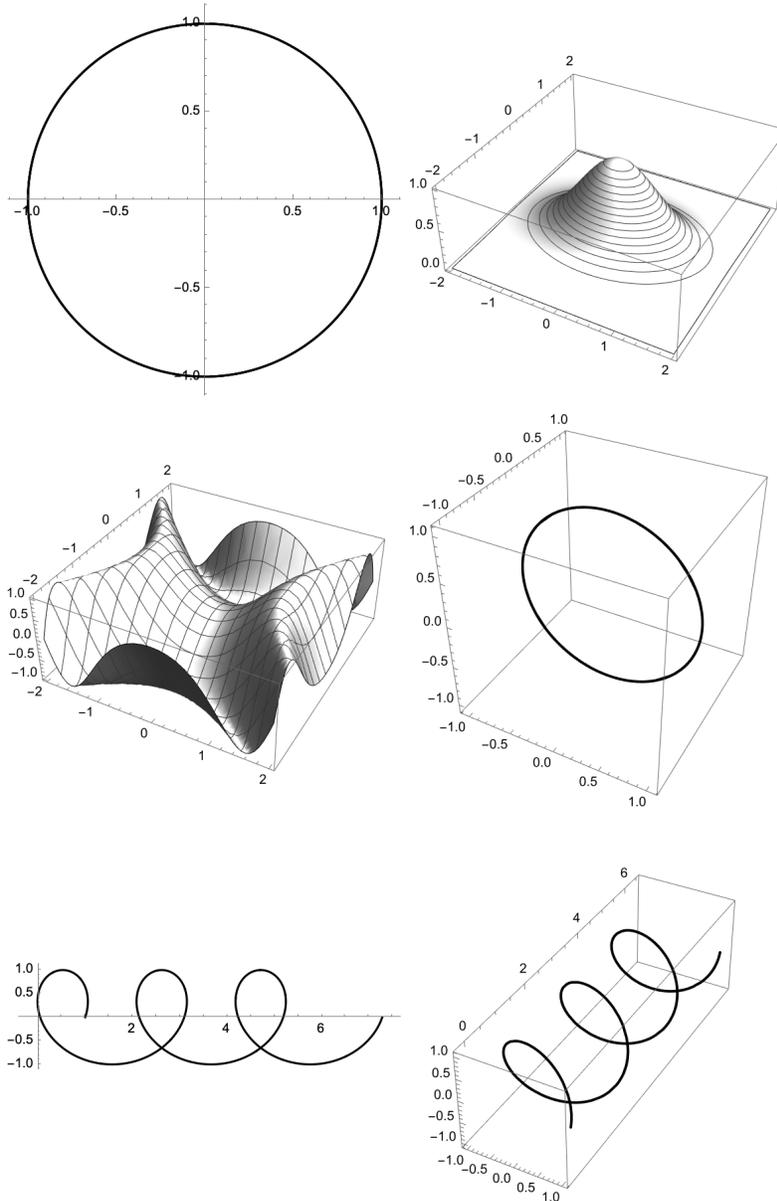


Figure 1: Some groovy pics