Prep for Quiz 2

Our second quiz will be this Wednesday, September 20. There will *certainly* be problems very much like 1 and 2 on the quiz; the probability is 100%. It's very likely there will be something very like 3 and/or 4; maybe a little different. Maybe there will be something else.

1. **Definitions** There will certainly be some version of this problem. You should be able to write down the following definitions (which are all in our text) verbatim - *don't feel the need to put these in your own words*!
   a. *Semi-conjugacy* of a function $f : S \to S$ with a function $g : T \to T$. Do write down the commutative diagram, as I think it helps keep things straight
   b. *Conjugacy* (You can simply state the extra assumption necessary to extend the definition of semi-conjugacy)
   c. *Criterion for a attractive orbit* (i.e. Theorem 2.25 in the "Critical orbits" section).
   d. *Sensitive dependence on initial conditions* (Essentially, Claim 2.44) but applied to an arbitrary function f, rather than the doubling map d.
2. **Apply conjugation**: Let $f(x) = x^2$ and let $\varphi(x) = 2x + 1$.
   a. Conjugate f by φ.
   b. Use your conjugation to describe the critical orbit of g.
   c. Given your knowledge of f, what can you say about the dynamics of g?
3. **The doubling map**: Let $d(x) = 2x \,\%\, 1$ denote the doubling map that maps $H : [0, 1)$ to itself.
   a. Find a point $x_0 \in H$ that has period 5 expressed as a binary expansion.
   b. Use the geometric series formula to express x_0 as a fraction.
4. **Applying the doubling map**: Continuing on with the previous problem, suppose I tell you that the doubling map is semi-conjugate to $f(x) = \frac{x^2}{2} + 3x - \frac{5}{2}$ via the semi-conjugacy $\varphi(x) = 4\cos(2\pi x) - 3$, i.e. $f \circ \varphi = \varphi \circ d$.
   a. Find a point of period 5 for f.
   b. What does your knowledge of d tell you about the general behavior of f?
5. **Finding a super-attractive parameter**: Consider the family $f_c(x) = x^2 - 4x + c$. Write down an equation that c must satisfy to ensure that f_c has a super-attractive orbit of period 3.