Complex variables - Review for exam II

1. Write down a careful statement of each of the following definitions or theorems:
   (a) Newton iteration
   (b) Principal logarithm (page 45)
   (c) Complex integral (page 53)
   (d) Cauchy's Theorem (Thm 4.9)
   (e) Cauchy's Integral Formula (Second version - Thm 4.13)
   (f) Fundamental theorem of algebra (Thm 5.7)
   (g) Liouville's Theorem (Cor 5.9)

2. Find all possible values of $(-4)^{-4}$ and $(-1/4)^{-1/4}$.

3. Find two branches of x^x such that such that $\text{im}(-1/2)^{-1/2} > 0$ for one branch and $\text{im}(-1/2)^{-1/2} < 0$ for the other.

4. Let $f(x) = x^2 + 4$. Perform two Newton iterates for f from $x_0 = i$. Two what complex number should the Newton iteration converge?

5. Suppose that f is entire, z_0 is a root of f, and N is the Newton's method iteration function for f. Show z_0 is a fixed point of N.

6. Compute
$$\int_\gamma \overline{z}\, dz,$$
where γ is the line segment from the origin to $1 + i$.

7. Let $C_r(z_0)$ be the standard, positively oriented parametrization of a circle of radius r centered about the point z_0. Show by direct computation that
$$\int_{C_r(z_0)} \frac{1}{z - z_0}\, dz = 2\pi i.$$
Next, let γ denote any positively oriented curve encircling z_0 exactly once. Explain how the previous computation, together with Cauchy's Integral Theorem imply that
$$\int_\gamma \frac{1}{z - z_0}\, dz = 2\pi i.$$

8. Let γ denote the circle of radius $3/2$ centered at the point i. Compute

$$\int_\gamma \frac{\sin(z)}{z^2(z^2-1)(z^2-4)}\, dz.$$

9. Let γ be a simple, closed loop in the top half of the plane enclosing the point i. Compute

$$\int_\gamma \frac{1}{z^2+1} dz.$$

Now, supposing that γ is a semi-circular arc with base on the real axis, compute

$$\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx.$$

10. Outline a procedure to compute

$$\int_{-\infty}^{\infty} \frac{1}{x^n+1} dx$$

when n is even. Why does this fail when n is odd?