Calc I - Review for exam 2

The second exam will be this Friday, October 19. We will discuss some of these problems in class on Wednesday, but you should work them all out to the best of your ability prior to that. Understanding the problems on this sheet will help you greatly on the exam.

1. Use the differentiation rules to find the derivative of each of the following functions.

   (a) $f(x) = x^2 + \frac{1}{x}$

   (b) $f(x) = e^x + \frac{1}{x}$

   (c) $f(x) = \frac{x^5}{\sin(x)}$

   (d) $f(x) = x^7\left(x^2 + x\right)$

   (e) $f(x) = \sqrt{\sin(x)} + \cos(x)$

   (f) $f(x) = \frac{1}{x^7}\cos\left(x^7\right)$

   (g) $f(x) = x^4\cos^2(x)$

   (h) $f(x) = \frac{e^x}{x^3}\left(x + \frac{1}{x}\right)$

   (i) $f(x) = \frac{4^{-x}}{x^4}$

   (j) $f(x) = \frac{e^{-x}\cos(x)}{\sin(\cos(x))}$

   (k) $f(x) = \tan(x^3) + x^3\sec(x)$

   (l) $f(x) = \arctan(x^3) + x^3\arcsin(x)$

2. Use the definition of the tangent function in terms of the sine and cosine to derive the fact that

$$\frac{d}{dx}\tan(x) = \sec^2(x).$$

   I suppose you might consider the same for the secant function.

3. In this problem, we're going to derive the fact that, if $f(x) = \ln(2x)$, then $f'(x) = 1/x$ using the fact that we know the inverse of f.

   (a) Starting from $y = \ln(2x)$, write the equation in exponential form.

   (b) Implicitly differentiate your equation from part (a) with respect to x.

   (c) Solve your equation from part (b) for y'.

   (d) Simplify, if necessary to show that $y' = 1/x$.

4. Compute the following limits using the fact that $\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1$.
   (a) $\lim_{x \to 0} \frac{\sin(3x)}{x}$
   (b) $\lim_{x \to 0} \frac{\sin(3x)}{\sin(2x)}$
   (c) $\lim_{x \to 0} \frac{1 - \cos^2(x)}{2x^2}$
5. Sketch the graphs of each of the following functions.
   (a) $f(x) = \sin(2x - \pi)$
   (b) $f(x) = 2\cos(\pi x) + 2$
6. In this problem, we'll use the pre-drawn axes in figure 1 to draw the graphs of $f(x) = \sin(2x)$ and $f'(x) = 2\cos(2x)$ and explore their relationship.
   (a) Sketch the graph of $f(x) = \sin(2x)$ on the top pair of axes.
   (b) Sketch the graph of $f'(x) = 2\cos(2x)$ on the bottom pair of axes.
   (c) Identify all points with horizontal tangent lines on the top graph. Projecting down, does the bottom graph cross the x axis at those points?
7. Let $f(x) = x\sin(x)$. Find an equation for the line tangent to the graph of f at the point $(\pi/2, f(\pi/2)$.
8. Use a linear approximation to find a good estimate to $\sqrt{101}$.
9. Let $f(x) = x^2 - 3$.
   (a) Find the corresponding Newton's method iteration function $N(x)$.
   (b) Perform two Newton iteration steps from the initial point $x_1 = 1$.
10. The graph on the left of figure 2 is of $y = x^3 - 3x^2 + 1$. For that graph:
    (a) Identify the maximum and minimum on the graph.
    (b) Find the exact coordinates of the maximum and minimum.
    (c) Identify the inflection point on the graph.
    (d) Find the exact coordinates of the inflection points.
11. Repeat the previous problem for $f(x) = 2xe^{-x}$, who's graph is shown on the right of figure 2.

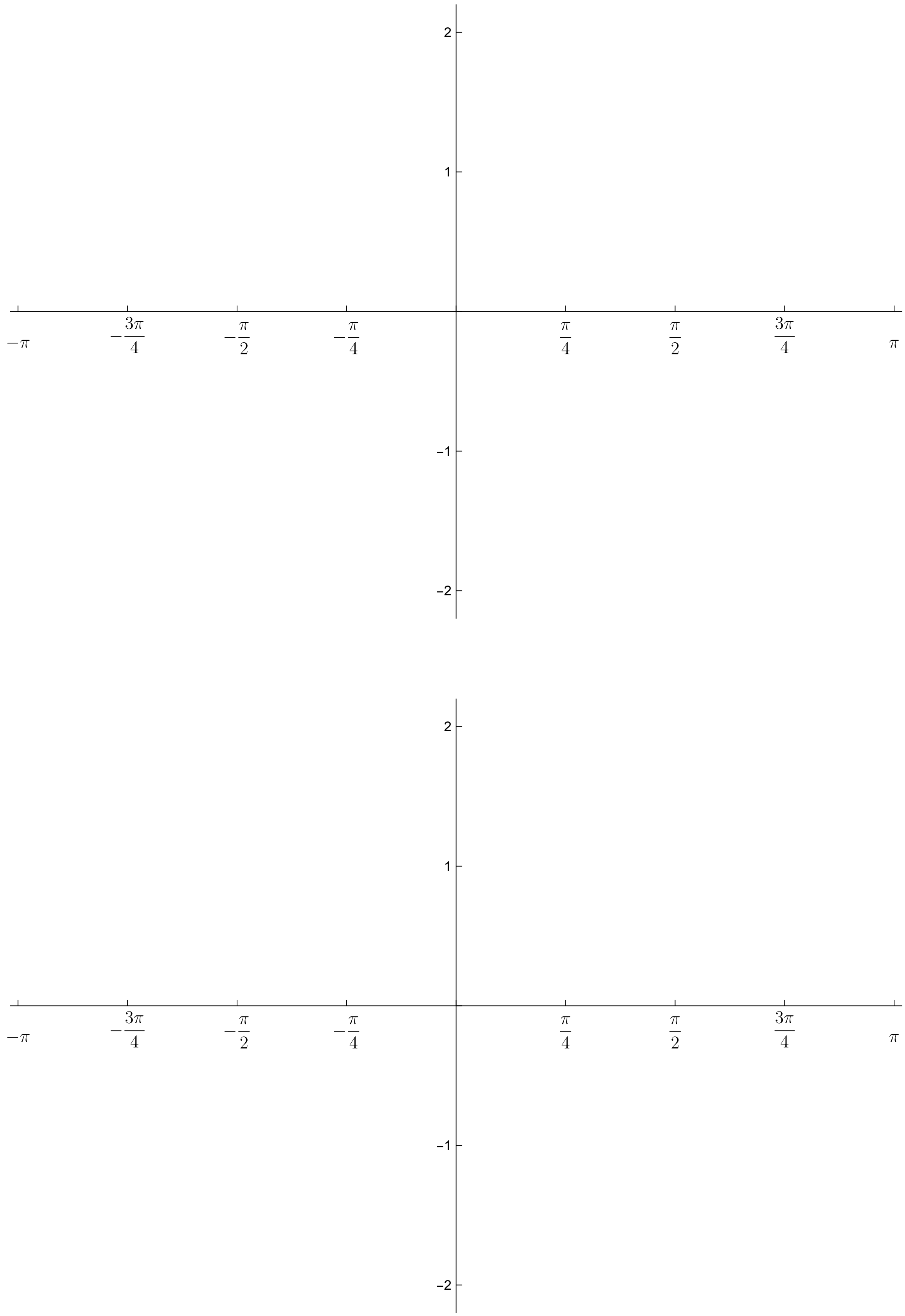

Figure 1: A pair of axes for trig functions

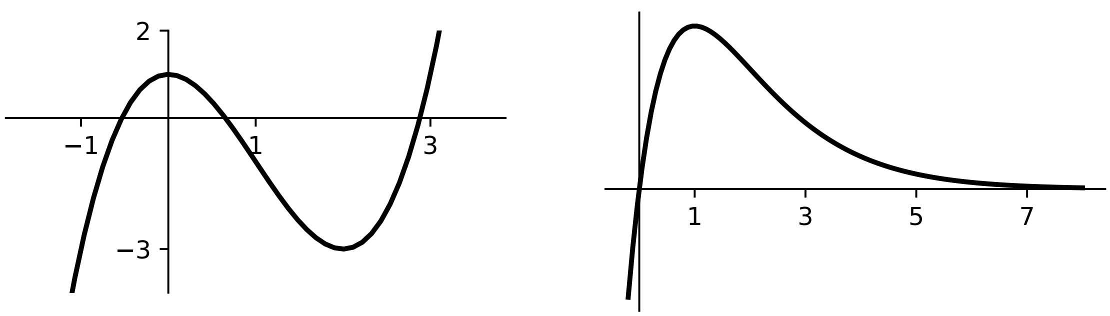

Figure 2: The graphs of $y = x^3 - 3x^2 + 1$ and $y = 2xe^{-x}$