Calc I - Review for exam I

The first exam will be this Friday, September 14. Here are some problems that might help.

Table 1: Usain Bolt's splits for the the 2008 Olympic 100m race

0-10m	10-20m	20-30m	30-40m	40-50m	50-60m	60-0m	70-80m	80-90m	90-100m	Total
1.85	1.02	0.91	0.87	0.85	0.82	0.82	0.82	0.83	0.90	9.69

1. Table 1 shows the splits for Usain Bolt's 100m race at the 2008 Olympics.
   (a) What was his average speed for the race in meters per second?
   (b) What is your best estimate his top speed?
2. We'd like to estimate
$$\lim_{t\to 0} \frac{7^t - 1}{t}$$
   so we generate the following table:

t	0.1	0.01	0.001	0.0001	0.00001
$f(t)$	2.14814	1.96497	1.94780	1.94610	1.94593

   Write down the value of the limit to as many decimal places that you are confident of.
3. Let's explore the derivative of $f(x) = 7^x$ using the difference quotient.
   (a) Write down the difference quotient for f.
   (b) Use a little algebra to separate the xs from the hs.
   (c) What's a good estimate to your derivative?
4. Let $f(x) = x^2 + 2x - 1$.
   (a) Sketch the graph of f, together with the line tangent to the graph at the point $(0, f(0))$.
   (b) Compute the slope of the secant line through the points $(0, f(0))$ and $(2, f(2))$. Draw this line on your graph from part (a).
   (c) Write down the difference quotient for f at 0, i.e. the slope of the secant line through the points $(0, f(0))$ and $(0 + h, f(0 + h))$.
   (d) Compute the limit as $h \to 0$ of your answer to part (c).
   (e) How does your answer to part (d) relate to your sketch from part (a)?

5. Let
$$f(x) = \frac{(5x-1)(x-3)}{x^2-9}.$$
Evaluate each of the following the limits.
   (a) $\lim_{x\to -1} f(x)$
   (b) $\lim_{x\to -3^-} f(x)$, $\lim_{x\to -3^+} f(x)$, and $\lim_{x\to -3} f(x)$
   (c) $\lim_{x\to 3} f(x)$
6. Figure 1 shows the complete graph of a function f; its domain is $(-1, 2]$.
   (a) What are $f(1.5)$ and $\lim_{x\to 1.5} f(x)$?
   (b) What are $\lim_{x\to 1^-} f(x)$, $\lim_{x\to 1^+} f(x)$, and $\lim_{x\to 1} f(x)$?
   (c) What can you say about $\lim_{x\to -1^+} f(x)$.
7. Find the derivatives of the following functions, *using the definition of the derivative.*
   (a) $f(x) = 2x^2 - 4x$
   (b) $f(x) = 1/\sqrt{x}$
   (c) $f(x) = x^5$
8. Find the derivatives of the following functions, using any technique you see fit.
   (a) $f(x) = 2x^2 - 4x$
   (b) $f(x) = 1/\sqrt{x}$
   (c) $f(x) = x^5$
   (d) $f(x) = x^5\left(x^2 - x - 1\right)$
   (e) $f(x) = \left(x^2 - x - 1\right)/x$
   (f) $f(x) = 2^x + 7^x + e^x + x^e$
9. Let $f(x) = x^2 - x$.
   (a) Write down the estimate for the slope of the graph of f at $x = 2$ yielded by the difference quotient using $h = 0.1$.
   (b) Write down an equation for the corresponding secant line.
   (c) Write down and equation for the precise tangent line at $x = 2$.
10. If f and g are differentiable function, prove that
$$\frac{d}{dx}(2f(x) + 3g(x)) = 2f'(x) + 3g'(x)$$
11. The complete graph of a function f is shown in figure 2; it consists of a line segment, a quarter-circle, and a semi-circle. Sketch the graph of f'.

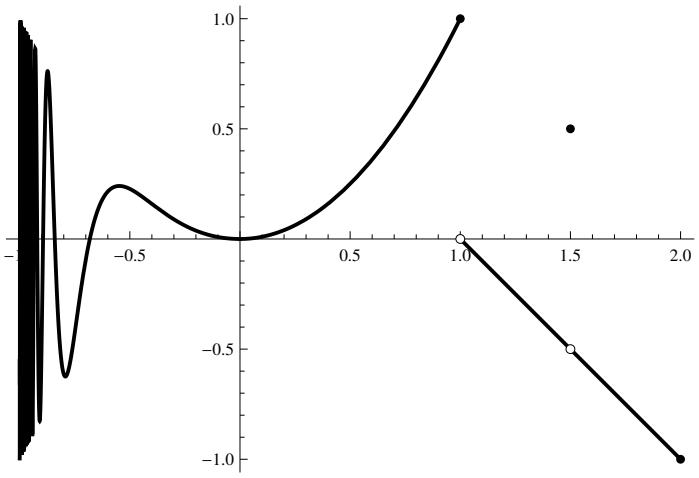

Figure 1: The graph for problem 6

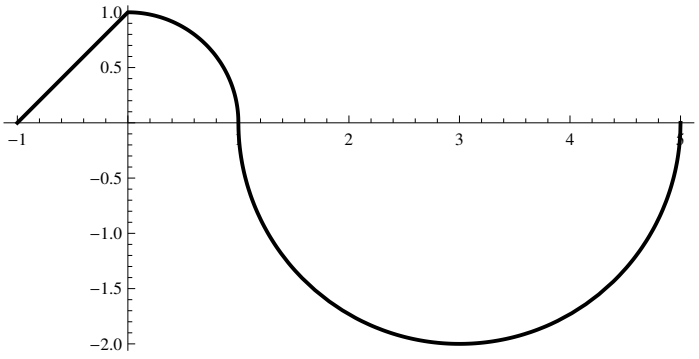

Figure 2: The graph for problem 11