Newton's method

with Mathematica

In this lab, we'll explore Newton's method (and functional iteration in general) with Mathematica.

First simple example

The problem: Find a decimal approximation to $\sqrt{2}$.

Solution: To recast this in terms of Newton's method, we need to find the positive root of $f(x) = x^2 - 2$. We can do this like so:

```
f[x_] = x^2 - 2;
n[x_] = x - f[x] / f'[x];
NestList[n, 1.0, 5]
```

Well, there's a lot packed into this little bit of code. The first two lines define the function of interest and then the corresponding Newton's method iteration function (which is auto computed in terms of `f`). That last line is where the action is. In general, `NestList[F,x0,m]` iterates the function `F` from the point `x0` a total of `m` times and returns the resulting list of numbers.

A trickier example

The problem: Find the roots of $f(x) = x^2 - 2 + \sin(22\,x)$.

Solution: I guess we just gotta change the function f. Let's try it.

```
f[x_] = x^2 - 2 + Sin[22 x];
n[x_] = x - f[x] / f'[x];
NestList[n, 1.0, 5]
```

Hmm... Better iterate a few more times. Try increasing the number of iterations.

When using Newton's method, it generally helps to look at a plot. Here's a quick plot of our function:

```
Plot[f[x], {x, -2, 2}]
```

So, I guess there's lots of roots - five positive and five negative. Since functions automatically map over lists, here's a slick way to find all the positive roots:

```
NestList[n, {1.2, 1.25, 1.4, 1.6, 1.65}, 5]
```

Note that the initial seed were guesstimates from the graph.

Problems

1. Use Newton's method to find all the negative roots of $f(x) = x^2 - 2 + \sin(22\,x)$.

2. Let $f(x) = x^3 - 3\,x - 3.7$.
 a) Plot the graph of f.
 b) Find the one real root of f.
 c) Perform the iteration from $x_0 = 0.0$.
Fire up our complex Newton method explorer here: https://goo.gl/4ZHsSy and generate an image of the basins of attraction for this polynomial. Do you see how this is related to the issue arising in part (c)?