

Chaos and Fractals: HW 1

This 20 point HW problem is due by next Tuesday, September 12. I expect it to be typed and you can email the PDF to me. I recommend that you type it using [L^AT_EX](#), though I'll consider other options if you're majoring in a discipline other than Mathematics. The easiest way to get started with [L^AT_EX](#) is probably using [Overleaf](#).

This problem is essentially problem 2.3.3 [from our textbook](#).

1. Suppose, that f is an *affine function*, which just means that it has the form

$$f(x) = ax + b,$$

where $a \neq 0$. Suppose also that $x_0 \in \mathbb{R}$ and let's consider the iterates

$$x_{n+1} = f(x_n).$$

- (a) Show that f has a unique fixed point iff $a \neq 1$. What if $a = 1$?
- (b) Suppose that $|a| < 1$. Show that the sequence of iterates converges to the fixed point of f .
- (c) Suppose that $|a| > 1$. Show that the sequence of iterates diverges.
- (d) What happens if $a = -1$?