Newton's Method

with Anaconda

In this lab, we'll explore Newton's method (and functional iteration in general) with Anaconda.

First simple example

The problem: Find a decimal approximation to $\sqrt{2}$.

Solution: To recast this in terms of Newton's method, we need to find the positive root of $f(x) = x^2 - 2$. We can do this like so:

In [ ]:
```
from sympy import *
x = var('x')

def f(x): return x**2 - 2
n = lambdify(x,x-f(x)/diff(f(x),x))
xi = 1.0
for i in range(8):
    xi = n(xi)
    print(xi)
```

Well, there's a few things going on here. The first two lines import all the functionality of the symbolic manipulation library SymPy into the global namespace and define `x` to be a symbolic variable. We then define the function `f` of interest and the corresponding Newton's method iteration function in terms of `f`. We then use a loop to iterate `f` eight times and print the results.

A trickier example

The problem: Find the roots of $f(x) = x^2 - 2 + \sin(22x)$.

Solution: I guess we just gotta change the function f. Let's try it.

In [ ]:
```
def f(x): return x**2 - 2 + sin(22*x)
n = lambdify(x,x-f(x)/diff(f(x),x))
xi = 1.0
for i in range(8):
    xi = n(xi)
    print(xi)
```

Hmm... Better iterate a few more times. Try increasing the number of iterations.

When using Newton's method, it generally helps to look at a plot. Here's a quick plot of our function:

In [ ]:
```
plot(f(x), (x,-2,2))
```

So, I guess there's lots of roots - five positive and five negative. Here's a way to find all the positive roots:

In [ ]:
```
n = lambdify(x,x-f(x)/diff(f(x),x))
for xi in [1.2, 1.25, 1.4, 1.6, 1.65]:
    while abs(f(xi))>10**-8: # Stupid
        xi = n(xi)
    print(xi)
```

Problems

1. Use Newton's method to find all the negative roots of $f(x) = x^2 - 2 + \sin(22x)$.
2. Let $f(x) = x^3 - 3x - 3.7$.

- Plot the graph of f.
- Find the one real root of f.
- Perform the iteration from Subscript[x, 0]=0.0.

Fire up our complex Newton method explorer here: https://goo.gl/4ZHsSy (https://goo.gl/4ZHsSy) and generate an image of the basins of attraction for this polynomial. Do you see how this is related to the issue arising in part (c)?