Review for our Complex QUAM

We'll have an assessment this coming Friday, December 2. It'll be somewhere between a quiz and and exam. Let's call it a QUAM.

1. Write down careful statements of the following definitions and theorems
   (a) Weierstrass M-test
   (b) The Cauchy integral theorem
   (c) The Cauchy integral formula
   (d) The classification of zeros theorem
   (e) The identity principle
2. Suppose that the series $\sum a_n(z-c)^n$ converges absolutely at $z_0 \neq c$ and suppose that $0 < R < |z_0 - c|$. Prove that the series converges uniformly on $\{z \in \mathbb{C} : |z - c| \leq R\}$.
3. Let γ be a simple, closed loop in the top half of the plane enclosing the point i. Compute
$$\int_\gamma \frac{1}{z^2+1} dz.$$
Now, supposing that γ is a semi-circular arc, compute
$$\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx.$$
4. Let γ be a simple, closed loop in the top half of the plane enclosing the point $1+i$. Compute
$$\int_\gamma \frac{1}{z^4+1} dz.$$
Now, supposing that γ is a semi-circular arc, compute
$$\int_{-\infty}^{\infty} \frac{1}{x^4+1} dx.$$
Note that $z^4+1 = (z-(1+i))(z-(1-i))(z-(-1+i))(z-(-1-i))$.
5. Find two Laurent series centered at zero for
$$f(z) = \frac{1}{z(z-4)}$$
and state their domains of convergence.