

Calc II - Review for exam II

The second exam will be this Friday, October 28. We will discuss some of these problems in class on Wednesday, but you should work them all out to the best of your ability prior to that. Understanding the problems on this sheet will help you greatly on the exam.

1. A 100 meter rope with linear density 1kg/m dangles over the edge of a cliff. How much work is required to pull the rope up to the top of the cliff?
2. Evaluate the following improper integrals, or state why they diverge.

(a) $\int_0^{\infty} e^{-2x} dx$

(b) $\int_1^{\infty} \frac{\ln(x)}{x^2} dx$

(c) $\int_1^{\infty} \frac{1}{x^3} dx$

3. Write down a complete sentence proving that the improper integral $\int_1^{\infty} e^{-x^2} dx$ converges.

Note: You may assume that $\int_1^{\infty} e^{-x} dx$ converges.

4. Suppose we spin the region under the graph of $f(x) = 5x^3(1-x)$ and over the x -axis around the x -axis. What is the volume of the resulting solid?
5. Suppose we spin the region under the graph of $f(x) = 1/x$ and over the semi-infinite interval $(1, \infty)$ around the x -axis. What is the volume of the resulting solid?
6. My one man tent is three feet high and is held up by two pieces of tent cord so that the base is half again as long as it is wide. What is the volume of that tent?
7. Use u -substitution to express the following normal integral as a standard normal integral:

$$\frac{1}{2\sqrt{2\pi}} \int_0^4 e^{-(x-3)^2/8} dx$$

8. Suppose that an unfair coin comes up heads $3/5$ of the time. Each time it comes up heads, we write down a one and each time it comes up tails, we write down a zero.
 - (a) Compute the mean and standard deviation associated with one such flip.
 - (b) Use a normal integral to estimate the probability that we get more than 125 heads in 200 flips.
9. I have a six sided die with two sides labeled 1, two sides labeled 2, and two sides labeled 3; thus, it generates a one, a two or a three with equal probability $1/3$. Now suppose I roll that die 150 times and add the resulting numbers.
 - (a) Write down a sum that shows the mean of one roll is $\mu = 2$. What is the mean associated with 150 rolls?
 - (b) Write down a sum that shows the variance of one roll is $\sigma^2 = 2/3$. What is the variance associated with 150 rolls?
 - (c) Write down a normal integral representing the probability that the sum total of my 150 rolls is more than 290 but not more than 310.

10. The object shown in the figure 1 on the right has horizontal slices that are equilateral triangles. The base has side length two, the top has side length one, and the height of the object is two. What is the volume of the object?

11. Suppose we wish to estimate

$$\int_0^3 \sqrt{x^3 + 1} dx$$

with a midpoint sum and we'd like our result to be within 0.0001 of the actual value.

- (a) Find an n large enough so that n terms will guarantee your estimate is within the desired accuracy.
- (b) Write down the resulting sum using summation notation.

Note that the graph of $f(x) = \sqrt{x^3 + 1}$ together with its second derivative is show in the figure below.

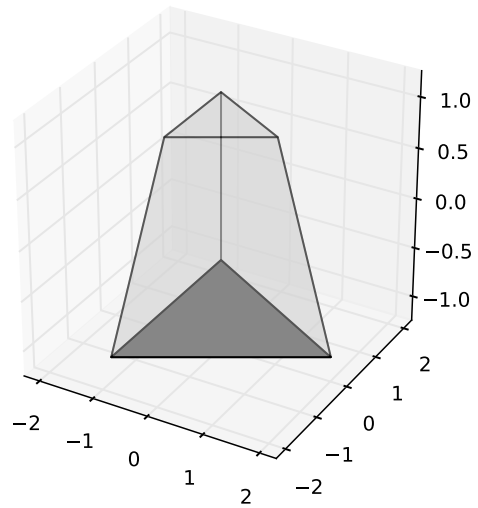
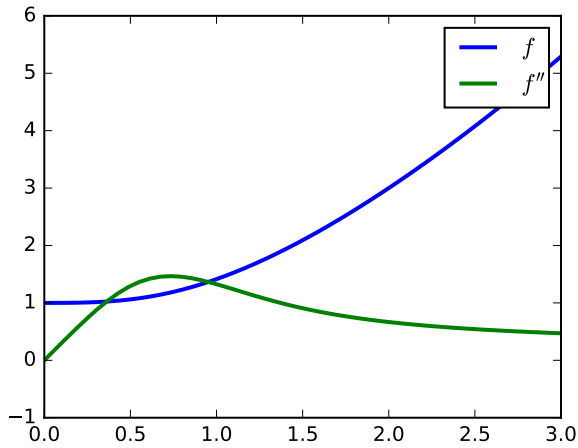


Figure 1: The graphs of $f(x) = \sqrt{x^3 + 1}$ and $f''(x)$ on the left and a pedestal on the right