Calc II - Problems off of past exams

Here are some of the problems we've had off of past exams. Together with our More Problems sheet, I recommend that you treat this like a review sheet.

Exam 1

1. Evaluate the following integrals using the technique indicated.
   (a) $\int x^3\sqrt{x^4+1}\,dx$ - u-subs
   (b) $\int x\ln(x)dx$ - by parts

4. (a) Use integration by parts to derive a reduction formula for the integral $\int_0^1 x^n e^x\,dx$.
   (b) Use your reduction formula to evaluate $\int_0^1 x^3 e^x\,dx$.

6. Evaluate the following integrals using any technique that you see fit.
   (a) $\int x^3\sqrt{x^2+1}\,dx$
   (b) $\int_0^\pi \cos^4(x)\,dx$
   (c) $\int_0^1 x^2\sqrt{1-x^2}\,dx$

Exam 2

2. Write down a complete sentence proving that the improper integral $\int_1^\infty \frac{1}{1+x^3}dx$ converges.

   Note: You may assume that $\int_1^\infty \frac{1}{x^3}dx$ converges.

5. Use u-substitution to express the following normal integral as a standard normal integral:

$$\frac{1}{4\sqrt{2\pi}}\int_{-1}^{2} e^{-(x-1)^2/32}\,dx$$

6. Suppose that an unfair coin comes up heads 2/5 of the time. Each time it comes up heads, we write down a one and each time it comes up tails, we write down a zero.
   (a) Compute the mean and standard deviation associated with one such flip.
   (b) Use a normal integral to estimate the probability that we get more than 490 heads in 1000 flips.

7. Suppose we wish to estimate

$$\int_0^2 \sin(x^2)dx$$

with a midpoint sum and we'd like our result to be within 0.0001 of the actual value.

(a) Find an n large enough so that n terms will guarantee your estimate is within the desired accuracy.

(b) Write down the resulting sum using summation notation.

Note that the graph of $f(x) = \sin(x^2)$ together with its second derivative is show in the figure 1. And don't forget that

$$E_n \leq M_2 \frac{(b-a)^3}{12\,n^2}!$$

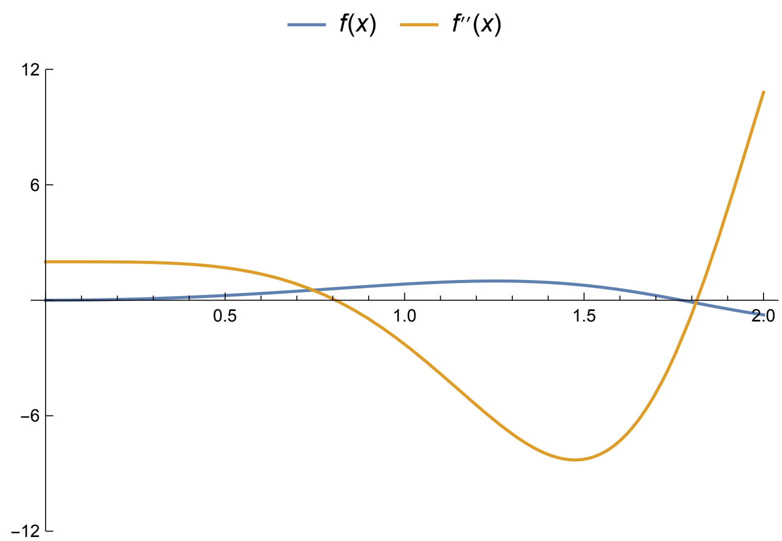

Figure 1: The graphs of $f(x) = \sin(x^2)$ and $f''(x)$

Exam3

1. Express $\sum_{n=2}^{\infty} 3\frac{4^{n+2}}{5^{n-3}}$ as a fraction.

2. Use the integral test to show that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges precisely when $p > 1$.

3. Suppose we'd like to approximate

$$\sum_{n=1}^{\infty} \frac{1}{n^4}$$

by truncating the sum to obtain a finite sum of the form

$$\sum_{n=1}^{N} \frac{1}{n^4}.$$

How large does N have to be to ensure that our approximation is within 0.0001 of the actual value?

4. Classify the following series as absolutely convergent, conditionally convergent, or divergent. Only an answer is required.

(a) $\sum (-1)^n \frac{n^4}{n^3 + 2n^2 + 1}$

(b) $\sum (-1)^n \frac{n^4}{n^5 + 2n^2 + 1}$

(c) $\sum (-1)^n \frac{n^4}{n^7 + 2n^2 + 1}$

5. Write down a couple of complete sentences using the comparison test to show that

$$\sum_{n=1}^{\infty} \frac{\cos(e^n)}{n^2 + 1}$$

converges absolutely.

6. Write down a couple complete sentences using the alternating series test to show that

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2 + 1}$$

converges conditionally.

7. Use the ratio test to determine whether

$$\sum_{n=0}^{\infty} \frac{n^{42}}{\sqrt{n!}}$$

converges or diverges.