

Problems - HW 4

Tuesday, September 8

1. Describe the geometric action of the following matrices using terms such as reflection, rotation, dilation, and skew.

(a) $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2. Let M denote the matrix

$$M = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix}$$

- (a) M represents a reflection about what line?
Your answer should be in terms of θ .
 - (b) Use this formulation to find a reflection about the line $y = \sqrt{3}x$.
3. Let A be the 2×2 matrix

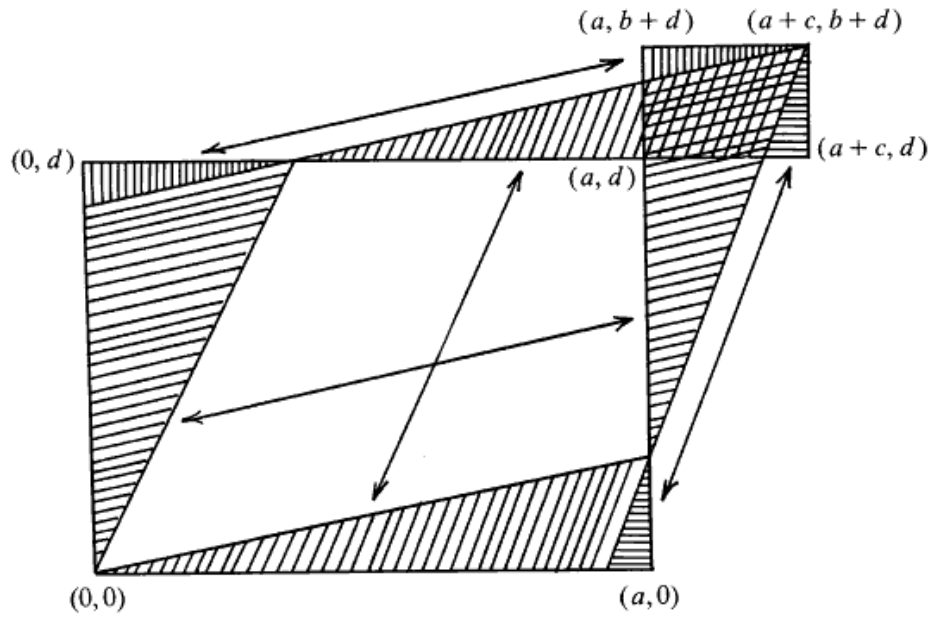
$$A = \begin{pmatrix} a & c \\ b & d \end{pmatrix}.$$

Use the figure on the reverse to explain why the area of the parallelogram spanned by the column vectors of A is $ad - bc$.

Acknowledgement: The figure comes from [math.stackexchange](https://math.stackexchange.com). Evidently, it originally appeared in the 1985 Math Mag.

Proof without words:

A 2×2 determinant is the area of a parallelogram



$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc = \left\| \begin{array}{|c|} \hline \square \\ \hline \end{array} \right\| - \left\| \begin{array}{|c|} \hline \square \\ \hline \end{array} \right\| = \left\| \begin{array}{|c|} \hline \square \\ \hline \end{array} \right\|$$