Problems on Riemann sums and error estimates

We've learned that there are a variety of sums (left and right Riemann sums, midpoint and trapezoidal sums, and even Simpson's sums) that can be used to approximate a definite integral. Our book does a nice job of summarizing these in the "Key idea 9" box on page 247 of section 5.5. The book then goes on to say in Theorem 43 on page 248 how the error in a trapezoidal sum can be bounded using the second derivative and the error in Simpson's rule can be bounded using the fourth derivative. I think it's worth knowing that there are similar (and simpler) formulae bounding the error of left and right Riemann sums. The following two theorems bound the error E_n is the error arising from estimating

$$\int_a^b f(x)dx$$

with a left or right Riemann sum.

Theorem 1: If f is monotone on $[a, b]$, then

$$E_n \leq |f(b) - f(a)| \frac{b-a}{n}.$$

A dynamic explanation of this inequality can be found on my Riemann error webpage. A more general version bounds the error on a Riemann sum for an arbitrary differentiable function, whether it is differentiable or not.

Theorem 2: If f is differentiable on $[a, b]$ and $|f(x)| \leq M$ for all $x \in [a, b]$, then

$$E_n \leq M \frac{(b-a)^2}{n}.$$

Problems

1. Suppose we wish to estimate

$$\int_0^2 2^{x^2}dx$$

with a right Riemann sum and we'd like our result to be within 0.0001 of the actual value.

   (a) Use Theorem 1 to find an n large enough so that n terms will guarantee your estimate is within the desired accuracy.

   (b) Write down the resulting sum using summation notation.

2. Suppose we wish to estimate

$$\int_0^2 x^2\cos(x^2)dx$$

with a left Riemann sum and we'd like our result to be within 0.0001 of the actual value.

   (a) Use Theorem 2 to find an n large enough so that n terms will guarantee your estimate is within the desired accuracy.

   (b) Write down the resulting sum using summation notation.